数学Ⅲ　定積分の部分積分 ① 課題　　(　　)年(　　)組(　　)番(　　　　　　)

1．次の定積分を求めよ。　Find the following definite integral.

$$\int_a^b f(x)\,g'(x)\,dx = \Big[f(x)\,g(x)\Big]_a^b - \int_a^b f'(x)\,g(x)\,dx$$

例題① $\displaystyle\int_0^{\pi} x\sin x\,dx = \int_0^{\pi} x(-\cos x)'\,dx$

$$= \Big[x(-\cos x)\Big]_0^{\pi} - \int_0^{\pi}(x)'(-\cos x)\,dx$$

$$= \Big[x(-\cos x)\Big]_0^{\pi} + \int_0^{\pi}\cos x\,dx$$

$$= \Big[x(-\cos x) + \sin x\Big]_0^{\pi}$$

$$= \{\pi(-\cos\pi) + \sin\pi\} - \{0(-\cos 0) + \sin 0\}$$

$$= \pi$$

問題① $\displaystyle\int_0^{\pi} x\cos x\,dx$

例題② $\displaystyle\int_1^e x\log x\,dx = \int_1^e \left(\frac{x^2}{2}\right)'\log x\,dx$

$$= \left[\frac{x^2}{2}\log x\right]_1^e - \int_1^e \frac{x^2}{2}(\log x)'\,dx$$

$$= \left[\frac{x^2}{2}\log x\right]_1^e - \int_1^e \frac{x^2}{2}\times\frac{1}{x}\,dx$$

$$= \left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]_1^e$$

$$= \left(\frac{e^2}{2}\log e - \frac{e^2}{4}\right) - \left(\frac{1^2}{2}\log 1 - \frac{1^2}{4}\right)$$

$$= \frac{e^2}{4} + \frac{1}{4}$$

問題② $\displaystyle\int_1^e \log x\,dx = \int_1^e (x)'\log x\,dx$

2．部分積分を利用して，次の定積分を求めよ。　Find the following definite integral using integrals by parts.

例題 $\displaystyle\int_{-1}^{1}(x+1)(x-1)^3\,dx$

$$= \int_{-1}^{1}(x+1)\left\{\frac{(x-1)^4}{4}\right\}'\,dx$$

$$= \left[(x+1)\times\frac{(x-1)^4}{4}\right]_{-1}^{1} - \int_{-1}^{1}\frac{(x-1)^4}{4}\,dx$$

$$= 0 - \left[\frac{(x-1)^5}{20}\right]_{-1}^{1}$$

$$= -\left(\frac{(1-1)^5}{20} - \frac{(-1-1)^5}{20}\right) = -\frac{8}{5}$$

問題① $\displaystyle\int_1^2 (x-1)(x-2)^3\,dx$

問題② $\displaystyle\int_1^2 (x-1)(x-2)^2\,dx$

数学Ⅲ　定積分の部分積分 ② 課題

(　　)年(　　)組(　　)番(　　　　　　　　　　)

1．次の定積分を求めよ。

$$\int_a^b f(x)\,g'(x)\,dx = \Big[f(x)\,g(x)\Big]_a^b - \int_a^b f'(x)\,g(x)\,dx$$

例題① $\displaystyle\int_0^{\pi} x\cos 2x\,dx = \int_0^{\pi} x\left(\frac{1}{2}\sin 2x\right)'dx$

$$= \left[\frac{1}{2}x\sin 2x\right]_0^{\pi} - \int_0^{\pi}(x)'\times\frac{1}{2}\sin 2x\,dx$$

$$= \left[\frac{1}{2}x\sin 2x\right]_0^{\pi} - \int_0^{\pi}\frac{1}{2}\sin 2x\,dx$$

$$= \left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right]_0^{\pi}$$

$$= \left(0+\frac{1}{4}\right) - \left(0+\frac{1}{4}\right) = 0$$

問題① $\displaystyle\int_0^{\pi} x\sin 2x\,dx$

例題② $\displaystyle\int_0^1 x\,e^{2x}\,dx = \int_0^1 x\left(\frac{1}{2}e^{2x}\right)'dx$

$$= \left[\frac{1}{2}x\,e^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x}(x)'\,dx$$

$$= \left[\frac{1}{2}x\,e^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x}\,dx$$

$$= \left[\frac{1}{2}x\,e^{2x} - \frac{1}{4}e^{2x}\right]_0^1$$

$$= \left(\frac{1}{2}e^2 - \frac{1}{4}e^2\right) - \left(0 - \frac{1}{4}\right)$$

$$= \frac{e^2}{4} + \frac{1}{4}$$

問題② $\displaystyle\int_0^1 x^2\,e^{2x}\,dx$

2．次の計算をせよ。

例題 $\Big[e^x\sin x\Big]_0^{\pi}$

$$= \left(e^{\pi}\sin\pi\right) - \left(e^0\sin 0\right) = 0$$

問題 $\Big[e^x\cos x\Big]_0^{\pi}$

3．部分積分を利用して，次の定積分を求めよ。

例題 $\displaystyle I = \int_0^{\pi} e^x\sin x\,dx$

$$I = \int_0^{\pi} e^x\sin x\,dx = \int_0^{\pi}(e^x)'\sin x\,dx$$

$$= \Big[e^x\sin x\Big]_0^{\pi} - \int_0^{\pi} e^x(\sin x)'\,dx$$

$$= \Big[e^x\sin x\Big]_0^{\pi} - \int_0^{\pi} e^x\cos x\,dx$$

$$= 0 - \left\{\Big[e^x\cos x\Big]_0^{\pi} - \int_0^{\pi} e^x(\cos x)'\,dx\right\}$$

$$= -\left\{\Big[e^x\cos x\Big]_0^{\pi} - \int_0^{\pi} e^x(-\sin x)\,dx\right\}$$

$$= -\Big[e^x\cos x\Big]_0^{\pi} + \int_0^{\pi} e^x\sin x\,dx$$

$$= e^{\pi} + 1 - I$$

したがって，　$2I = e^{\pi} + 1$

よって　$I = \dfrac{1}{2}e^{\pi} + \dfrac{1}{2}$

問題 $\displaystyle J = \int_0^{\pi} e^x\cos x\,dx$

数学Ⅲ　定積分の部分積分 ③ 課題

(　　)年(　　)組(　　)番(　　　　　　　　　　)

1．次の定積分を求めよ。

例題①　$\displaystyle\int_0^{\pi} x\cos x\,dx = \int_0^{\pi} x(\sin x)'\,dx$

$\displaystyle= \Big[x\sin x\Big]_0^{\pi} - \int_0^{\pi}(x)'\sin x\,dx$

$\displaystyle= \Big[x\sin x\Big]_0^{\pi} - \int_0^{\pi}\sin x\,dx$

$\displaystyle= \Big[x\sin x + \cos x\Big]_0^{\pi}$

$= -2$

問題①　$\displaystyle\int_0^{\pi} x\sin x\,dx$

例題②　$\displaystyle\int_0^{\pi} x^2\sin x\,dx = \int_0^{\pi} x^2(-\cos x)'\,dx$

$\displaystyle= \Big[-x^2\cos x\Big]_0^{\pi} - \int_0^{\pi}(x^2)'(-\cos x)\,dx$

$\displaystyle= \Big[-x^2\cos x\Big]_0^{\pi} + 2\int_0^{\pi} x\cos x\,dx$

$\displaystyle= \Big[-x^2\cos x + 2x\sin x + 2\cos x\Big]_0^{\pi}$

$= \pi^2 - 4$

問題②　$\displaystyle\int_0^{\pi} x^2\cos x\,dx$

2．次の定積分を求めよ。

例題①　$\displaystyle\int_0^{1} x e^{-x}\,dx = \int_0^{1} x(-e^{-x})'\,dx$

$\displaystyle= \Big[-xe^{-x}\Big]_0^{1} - \int_0^{1}(x)'(-e^{-x})\,dx$

$\displaystyle= \Big[-xe^{-x}\Big]_0^{1} + \int_0^{1} e^{-x}\,dx$

$\displaystyle= \Big[-xe^{-x} - e^{-x}\Big]_0^{1}$

$= (-e^{-1} - e^{-1}) - (-1) = -2e^{-1} - 1$

問題①　$\displaystyle\int_0^{1} x e^{x}\,dx$

例題②　$\displaystyle\int_0^{1} x^2 e^{-x}\,dx$

$\displaystyle= \Big[-x^2 e^{-x}\Big]_0^{1} - \int_0^{1}(x^2)'(-e^{-x})\,dx$

$\displaystyle= \Big[-x^2 e^{-x}\Big]_0^{1} + 2\int_0^{1} x e^{-x}\,dx$

$\displaystyle= \Big[-x^2 e^{-x} - 2xe^{-x} - 2e^{-x}\Big]_0^{1}$

$= (-5e^{-1}) - (-2) = -5e^{-1} + 2$

問題②　$\displaystyle\int_0^{1} x^2 e^{x}\,dx$

数学Ⅲ　定積分の部分積分(対数関数・指数関数)　課題　　(　　)年(　　)組(　　)番(　　　　　　　　)

1．次の不定積分を求めよ。

例題①　$\displaystyle\int \frac{(\log x)^2}{x}\,dx$　※置換積分

$\log x = t$ とおくと　$\dfrac{dt}{dx} = \dfrac{1}{x}$ ，$\dfrac{1}{x}\,dx = dt$

$$\int \frac{(\log x)^2}{x}\,dx$$

$$= \int t^2\,dt = \frac{1}{3}t^3 + C = \frac{1}{3}(\log x)^3 + C$$

問題①　$\displaystyle\int \frac{\log x}{x}\,dx$　※置換積分

例題②　$\displaystyle\int \log(x+1)\,dx = \int (x+1)'\log(x+1)\,dx$

$$= (x+1)\log(x+1) - \int (x+1)\{\log(x+1)\}'\,dx$$

$$= (x+1)\log(x+1) - \int (x+1)\left(\frac{1}{x+1}\right)dx$$

$$= (x+1)\log(x+1) - \int dx$$

$$= (x+1)\log(x+1) - x + C$$

問題②　$\displaystyle\int \log x\,dx = \int (x)'\log x\,dx$

例題③　$\displaystyle\int x e^{-2x}\,dx = \int x\left(-\frac{1}{2}e^{-2x}\right)'dx$

$$= -\frac{1}{2}xe^{-2x} - \int (x)'\left(-\frac{1}{2}e^{-2x}\right)dx$$

$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$

$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$$

問題③　$\displaystyle\int x e^{2x}\,dx = \int x\left(\frac{1}{2}e^{2x}\right)'dx$

2．次の定積分を求めよ。

例題①　$\displaystyle\int_1^e \frac{(\log x)^2}{x}\,dx$　※置換積分

$$= \left[\frac{1}{3}(\log x)^3\right]_1^e$$

$$= \left\{\frac{1}{3}(\log e)^3\right\} - \left\{\frac{1}{3}(\log 1)^3\right\} = \frac{1}{3}$$

問題①　$\displaystyle\int_1^e \frac{\log x}{x}\,dx$　※置換積分

例題②　$\displaystyle\int_1^3 \log(x+1)\,dx$

$$= \Big[(x+1)\log(x+1) - x\Big]_1^3$$

$$= (4\log 4 - 4) - (2\log 2 - 2) = 6\log 2 - 2$$

問題②　$\displaystyle\int_2^4 \log x\,dx$

例題③　$\displaystyle\int_0^2 x e^{-x}\,dx = \int x(-e^{-x})'\,dx$

$$= \Big[-xe^{-x}\Big]_0^2 - \int_0^2 (x)'(-e^{-x})\,dx$$

$$= \Big[-xe^{-x}\Big]_0^2 + \int_0^2 e^{-x}\,dx$$

$$= \Big[-xe^{-x} - e^{-x}\Big]_0^2$$

$$= \left(-2e^{-2} - e^{-2}\right) - \left(-0e^{-0} - e^{-0}\right)$$

$$= -3e^{-2} + 1$$

問題③　$\displaystyle\int_0^1 x e^{x}\,dx$

数学Ⅲ　定積分の部分積分(三角関数)　課題　　(　　)年(　　)組(　　)番(　　　　　　　　)

1．次の不定積分を求めよ。

例題①　$\displaystyle\int x\sin 2x\,dx = \int x\left(-\frac{1}{2}\cos 2x\right)'dx$

$\displaystyle = x\left(-\frac{1}{2}\cos 2x\right) - \int (x)'\left(-\frac{1}{2}\cos 2x\right)dx$

$\displaystyle = x\left(-\frac{1}{2}\cos 2x\right) - \int \left(-\frac{1}{2}\sin 2x\right)dx$

$\displaystyle = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$

問題①　$\displaystyle\int x\cos 2x\,dx$

例題②　$\displaystyle\int x^2\cos 2x\,dx = \int x^2\left(\frac{1}{2}\sin 2x\right)'dx$

$\displaystyle = x^2\left(\frac{1}{2}\sin 2x\right) - \int (x^2)'\left(\frac{1}{2}\sin 2x\right)dx$

$\displaystyle = x^2\left(\frac{1}{2}\sin 2x\right) - \int x\sin 2x\,dx$

$\displaystyle = \frac{1}{2}x^2\sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x + C$

問題②　$\displaystyle\int x^2\sin 2x\,dx$

2．次の定積分を求めよ。

例題　$\displaystyle\int_0^{\frac{\pi}{2}} x\sin 2x\,dx$

$\displaystyle = \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$

$\displaystyle = \left(-\frac{\pi}{4}\cos\pi + \frac{1}{4}\sin\pi\right) - \left(0 + \frac{1}{4}\sin 0\right)$

$\displaystyle = \left(\frac{\pi}{4} + 0\right) - (0 + 0) = \frac{\pi}{4}$

問題　$\displaystyle\int_0^{\frac{\pi}{2}} x\cos 2x\,dx$

3．次の定積分を求めよ。

例題　$\displaystyle\int_0^{\frac{\pi}{2}} x^2\sin 2x\,dx$

$\displaystyle = \left[-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{2}}$

$\displaystyle = \left(-\frac{\pi^2}{8}\cos\pi + \frac{\pi}{4}\sin\pi + \frac{1}{4}\cos\pi\right)$

$\displaystyle -\left(0\cos 0 + 0\sin 0 + \frac{1}{4}\cos 0\right)$

$\displaystyle = \left(\frac{\pi^2}{8} + 0 - \frac{1}{4}\right) - \left(0 + 0 + \frac{1}{4}\right)$

$\displaystyle = \frac{\pi^2}{8} - \frac{1}{2}$

問題　$\displaystyle\int_0^{\frac{\pi}{2}} x^2\cos 2x\,dx$

4．次の定積分を求めよ。

例題　$\displaystyle\int_0^{\frac{\pi}{2}} (x+1)\cos x\,dx$

$\displaystyle = \int_0^{\frac{\pi}{2}} (x+1)(\sin x)'\,dx$

$\displaystyle = \Big[(x+1)\sin x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}\sin x\,dx$

$\displaystyle = \Big[(x+1)\sin x + \cos x\Big]_0^{\frac{\pi}{2}}$

$\displaystyle = \left\{\left(\frac{\pi}{2} + 1\right)\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right\}$

$\displaystyle -\left\{(0+1)\sin 0 + \cos 0\right\}$

$\displaystyle = \left(\frac{\pi}{2} + 1\right) - 1 = \frac{\pi}{2}$

問題　$\displaystyle\int_0^{\frac{\pi}{2}} (x+1)\sin x\,dx$