

つぎていせきぶんもと

1. 次の定積分を求めよ。

Find the following definite integral.

つぎていせきぶんもと

2. 次の定積分を求めよ。

Find the following definite integral.

れいだい

例題

$$\int_0^2 x(2-x)^2 dx$$
$$t = 2 - x \text{ とおくと } \frac{dt}{dx} = -1, \quad dx = (-1) dt$$
$$\int_0^2 x(2-x)^2 dx \qquad \left. \begin{matrix} x \\ t \end{matrix} \right\| \begin{matrix} 0 & 2 \\ 2 & 0 \end{matrix}$$
$$= \int_2^0 (2-t) t^2 (-1) dt = \int_2^0 (t^3 - 2t^2) dt$$
$$= \left[ \frac{t^4}{4} - \frac{2t^3}{3} \right]_2^0$$
$$= \left( \frac{0^4}{4} - \frac{2 \times 0^3}{3} \right) - \left( \frac{2^4}{4} - \frac{2 \times 2^3}{3} \right) = -\frac{4}{3}$$

もんだい

問題

$$\int_0^3 x(3-x)^2 dx$$

もんだい

問題

$$\int_0^1 x(x-1)^2 dx$$

れいだい

例題

$$\int_0^3 x\sqrt{x+1} dx$$
$$t = \sqrt{x+1} \text{ とおくと } x = t^2 - 1$$
$$\frac{dx}{dt} = 2t, \quad dx = 2t dt \qquad \left. \begin{matrix} x \\ t \end{matrix} \right\| \begin{matrix} 0 & 2 \\ 2 & 0 \end{matrix}$$
$$\int_0^3 x\sqrt{x+1} dx$$
$$= \int_1^2 (t^2 - 1) t \times 2t dt = \int_1^2 (2t^4 - 2t^2) dt$$
$$= \left[ \frac{2}{5} t^5 - \frac{2}{3} t^3 \right]_1^2$$
$$= \left( \frac{64}{5} - \frac{16}{3} \right) - \left( \frac{2}{5} - \frac{2}{3} \right) = \frac{116}{15}$$

もんだい

問題

$$\int_2^3 x\sqrt{x-2} dx$$

もんだい

問題

$$\int_0^1 x\sqrt{1-x} dx$$

1. 次の定積分を計算せよ。

れいだい

例題

$$\int_0^1 x(1-x)^3 dx$$
$$t = 1 - x \text{とおくと } \frac{dt}{dx} = -1, \quad dx = (-1) dt$$
$$\int_0^1 x(1-x)^3 dx \qquad \frac{x}{t} \parallel \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}$$
$$= \int_1^0 (1-t)^3 (-1) dt = \int_1^0 (1-t)^3 dt$$
$$= \left[ -\frac{1}{4}(1-t)^4 \right]_1^0$$
$$= \left( -\frac{1}{4} \right) - \left( -\frac{1}{4} \right) = 0$$

もんだい

問題

$$\int_0^2 x(2-x)^4 dx$$

もんだい

問題

$$\int_0^1 x(1-x)^5 dx$$

2. 次の定積分を計算せよ。

れいだい

例題

$$\int_0^5 \frac{1}{\sqrt{3x+1}} dx$$
$$t = \sqrt{3x+1} \text{とおくと } x = \frac{t^2-1}{3}$$
$$\frac{dx}{dt} = \frac{2}{3}t, \quad dx = \frac{2}{3}t dt \qquad \frac{x}{t} \parallel \begin{array}{cc} 0 & 5 \\ 1 & 4 \end{array}$$
$$\int_0^5 \frac{1}{\sqrt{3x+1}} dx = \int_1^4 \frac{1}{t} \times t dt$$
$$= \frac{2}{3} \left[ t \right]_1^4 = \frac{2}{3} (4-1) = 2$$

もんだい

問題

$$\int_0^4 \frac{1}{\sqrt{2x+1}} dx$$

れいだい

例題

$$\int_0^3 \sqrt{4-x} dx$$
$$t = \sqrt{4-x} \text{とおくと } x = 4-t^2$$
$$\frac{dx}{dt} = -2t, \quad dx = -2t dt \qquad \frac{x}{t} \parallel \begin{array}{cc} 0 & 3 \\ 2 & 1 \end{array}$$
$$\int_0^3 \sqrt{4-x} dx = \int_2^1 t \times (-2t) dt$$
$$= -2 \int_2^1 t^2 dt = -2 \left[ \frac{t^3}{3} \right]_2^1$$
$$= -2 \left( \frac{1}{3} - \frac{8}{3} \right) = \frac{14}{3}$$

もんだい

問題

$$\int_1^5 \sqrt{5-x} dx$$

1. 次の定積分を計算せよ。

れいだい  
例題

$$\int_0^1 (2x + 1)^3 dx$$

$t = 2x + 1$  とおくと

$$\frac{dt}{dx} = 2, \quad dx = \frac{1}{2} dt$$

$\frac{x}{t}$	$\parallel$	0	1
$t$	$\parallel$	1	3

$$\int_0^1 (2x + 1)^3 dx = \int_1^3 t^3 \times \frac{1}{2} dt$$
$$= \left[ \frac{t^4}{8} \right]_1^3 = \frac{81}{8} - \frac{1}{8} = 10$$

れいだい  
例題

$$\int_0^2 (3x - 1)^3 dx$$

れいだい  
例題

$$\int_0^1 \frac{2x}{x^2 + 1} dx$$
$$\int_0^1 \frac{2x}{x^2 + 1} dx = \int_0^1 \frac{(x^2 + 1)'}{x^2 + 1} dx$$
$$= \left[ \log(x^2 + 1) \right]_0^1 = \log 2 - \log 1 = \log 2$$

もんだい  
問題

$$\int_0^2 \frac{2x}{x^2 + 4} dx$$

もんだい  
問題

$$\int_0^1 \frac{3x^2}{x^3 + 1} dx$$

2. 次の定積分を計算せよ。

れいだい  
例題

$$\int_0^1 \frac{x dx}{\sqrt{x^2 + 1}}$$

$t = \sqrt{x^2 + 1}$  とおくと

$$\frac{dt}{dx} = \frac{x}{\sqrt{x^2 + 1}}, \quad dt = \frac{x dx}{\sqrt{x^2 + 1}}$$

$\frac{x}{t}$	$\parallel$	0	1
$t$	$\parallel$	1	$\sqrt{2}$

$$\int_0^1 \frac{x dx}{\sqrt{x^2 + 1}} = \int_1^{\sqrt{2}} dt = \left[ t \right]_1^{\sqrt{2}} = \sqrt{2} - 1$$

もんだい  
問題

$$\int_0^4 \frac{x dx}{\sqrt{x^2 + 4}}$$

れいだい  
例題

$$\int_0^1 x\sqrt{x^2 + 3} dx$$

$t = \sqrt{x^2 + 3}$  とおくと ,  $x^2 = t^2 - 3$

$$2x \frac{dx}{dt} = 2t, \quad x dx = t dt$$

$\frac{x}{t}$	$\parallel$	0	1
$t$	$\parallel$	$\sqrt{3}$	2

$x = 0$  のとき  $t = \sqrt{3}$  ,  $x = 1$  のとき  $t = 2$

$$\int_0^1 x\sqrt{x^2 + 1} dx = \int_{\sqrt{3}}^2 t \times t dt$$
$$= \int_{\sqrt{3}}^2 t^2 dt = \left[ \frac{t^3}{3} \right]_{\sqrt{3}}^2 = \frac{8}{3} - \sqrt{3}$$

もんだい  
問題

$$\int_0^1 x\sqrt{x^2 + 1} dx$$

1. 次の定積分を計算せよ。

例題

$$\int_0^3 \sqrt{9-x^2} \, dx$$

$x = 3 \sin t$  とおくと

$dx = 3 \cos t \, dt$

この範囲では  $\cos t \geq 0$  である。

$$\begin{aligned} \sqrt{9-x^2} &= \sqrt{9-(3 \sin t)^2} = \sqrt{9(1-\sin^2 t)} \\ &= \sqrt{9 \cos^2 t} = 3 \cos t \end{aligned}$$

$$\int_0^3 \sqrt{9-x^2} \, dx = \int_0^{\frac{\pi}{2}} (3 \cos t) \times 3 \cos t \, dt$$

$$= 9 \int_0^{\frac{\pi}{2}} \cos^2 t \, dt = 9 \int_0^{\frac{\pi}{2}} \left( \frac{1+\cos 2t}{2} \right) dt$$

$$= \frac{9}{2} \left[ t + \frac{1}{2} \sin 2t \right]_0^{\frac{\pi}{2}}$$

$$= \frac{9}{2} \left\{ \left( \frac{\pi}{2} + \frac{1}{2} \sin \pi \right) - \left( 0 + \frac{1}{2} \sin 0 \right) \right\}$$

$$= \frac{9\pi}{4}$$

問題

$$\int_{-1}^1 \sqrt{1-x^2} \, dx$$

問題

$$\int_0^1 \sqrt{4-x^2} \, dx$$

2. 次の定積分を計算せよ。

例題

$$\int_0^3 \frac{dx}{x^2+9}$$

$x = 3 \tan t$  とおくと

$dx = \frac{3 \, dt}{\cos^2 t}$

$$\int_0^3 \frac{dx}{x^2+9} = \int_0^{\frac{\pi}{4}} \frac{1}{(3 \tan t)^2+9} \times \frac{3 \, dt}{\cos^2 t}$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{9(\tan^2 t+1)} \times \frac{3 \, dt}{\cos^2 t}$$

$$= \int_0^{\frac{\pi}{4}} \frac{\cos^2 t}{9} \times \frac{3 \, dt}{\cos^2 t} = \frac{1}{3} \int_0^{\frac{\pi}{4}} dt$$

$$= \frac{1}{3} \left[ t \right]_0^{\frac{\pi}{4}} = \frac{1}{3} \left( \frac{\pi}{4} - 0 \right)$$

$$= \frac{\pi}{12}$$

問題

$$\int_0^1 \frac{dx}{x^2+1}$$

問題

$$\int_{-2}^2 \frac{dx}{x^2+4}$$

1. 次の定積分を計算せよ。

例題

$$\int_0^3 \frac{1}{\sqrt{9-x^2}} dx$$

$$x = 3 \sin \theta \quad \text{とおくと}$$
$$dx = 3 \cos \theta \, d\theta$$
$$\begin{array}{c|c} x & 0 & 3 \\ \hline & 0 & \frac{3}{2} \end{array}$$

この範囲では  $\cos \theta \geq 0$  である。

$$\sqrt{9-x^2} = \sqrt{9-(3 \sin \theta)^2} = \sqrt{9(1-\sin^2 \theta)}$$
$$= \sqrt{9 \cos^2 \theta} = 3 \cos \theta$$

$$\int_0^3 \frac{1}{\sqrt{9-x^2}} dx$$
$$= \int_0^{\frac{\pi}{2}} \frac{1}{3 \cos \theta} \times 3 \cos \theta \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} d\theta = \left[ \theta \right]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

問題

$$\int_0^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}} dx$$

問題

$$\int_0^2 \frac{1}{\sqrt{16-x^2}} dx$$

2. 次の定積分を計算せよ。

例題

$$\int_0^{\sqrt{3}} \frac{dx}{(x^2+3)^2}$$

$$x = \sqrt{3} \tan \theta \quad \text{とおくと}$$
$$dx = \frac{\sqrt{3} \, d\theta}{\cos^2 \theta}$$
$$\begin{array}{c|c} x & 0 & \sqrt{3} \\ \hline & 0 & \frac{\pi}{4} \end{array}$$

$$\int_0^{\sqrt{3}} \frac{dx}{(x^2+3)^2}$$
$$= \int_0^{\frac{\pi}{4}} \frac{1}{(3 \tan^2 \theta + 3)^2} \times \frac{\sqrt{3} \, d\theta}{\cos^2 \theta}$$
$$= \int_0^{\frac{\pi}{4}} \frac{1}{9 (\tan^2 \theta + 1)^2} \times \frac{\sqrt{3} \, d\theta}{\cos^2 \theta}$$
$$= \int_0^{\frac{\pi}{4}} \frac{\cos^4 \theta}{9} \times \frac{\sqrt{3} \, d\theta}{\cos^2 \theta}$$
$$= \frac{\sqrt{3}}{9} \int_0^{\frac{\pi}{4}} \cos^2 \theta \, d\theta$$
$$= \frac{\sqrt{3}}{9} \int_0^{\frac{\pi}{4}} \frac{1 + \cos 2\theta}{2} \, d\theta$$
$$= \frac{\sqrt{3}}{18} \left[ \theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{4}} = \frac{\sqrt{3}}{72} \left( \frac{\pi}{4} + 2 \right)$$
$$= \frac{\sqrt{3}}{18} \left\{ \left( \frac{\pi}{4} + \frac{1}{2} \sin \frac{\pi}{2} \right) - \left( 0 + \frac{1}{2} \sin 0 \right) \right\}$$

問題

$$\int_0^1 \frac{dx}{(x^2+1)^2}$$

1. 次の定積分を計算せよ。

例題

$$\int_0^2 x^2 \sqrt{4 - x^2} \, dx$$

$x = 2 \sin \theta$  とおくと

$$dx = 2 \cos \theta \, d\theta$$

この範囲では  $\cos \theta \geq 0$  である。

$$\begin{aligned} \sqrt{4 - x^2} &= \sqrt{4 - (2 \sin \theta)^2} = \sqrt{4 (1 - \sin^2 \theta)} \\ &= \sqrt{4 \cos^2 \theta} = 2 \cos \theta \end{aligned}$$

$$\begin{aligned} \int_0^2 x^2 \sqrt{4 - x^2} \, dx &= \int_0^{\frac{\pi}{2}} (2 \sin \theta)^2 \times 2 \cos \theta \times 2 \cos \theta \, d\theta \\ &= 16 \int_0^{\frac{\pi}{2}} \sin^2 \theta \cos^2 \theta \, d\theta \\ &= 16 \int_0^{\frac{\pi}{2}} \left( \frac{1}{2} \sin 2\theta \right)^2 d\theta \\ &= 4 \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2\theta}{2} d\theta \\ &= 2 \left[ \theta - \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{2}} \\ &= 2 \left\{ \left( \frac{\pi}{2} - \frac{1}{2} \sin \pi \right) - \left( 0 - \frac{1}{2} \sin 0 \right) \right\} = \end{aligned}$$

問題

$$\int_0^1 x^2 \sqrt{1 - x^2} \, dx$$

2. 次の定積分を計算せよ。

例題

$$\int_0^1 \frac{dx}{\sqrt{x^2 + 3}}$$

$x = \sqrt{3} \tan \theta$  とおくと

$$dx = \frac{\sqrt{3} d\theta}{\cos^2 \theta}$$

$$\frac{1}{\sqrt{x^2 + 3}} = \frac{1}{\sqrt{3 (1 + \tan^2 \theta)}} = \frac{\cos \theta}{\sqrt{3}}$$

$$\begin{aligned} \int_0^1 \frac{dx}{\sqrt{x^2 + 3}} &= \int_0^{\frac{\pi}{6}} \frac{\cos \theta}{\sqrt{3}} \times \frac{\sqrt{3} d\theta}{\cos^2 \theta} \\ &= \int_0^{\frac{\pi}{6}} \frac{d\theta}{\cos \theta} = \frac{1}{2} \log 3 \end{aligned}$$

問題

$$\int_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 3}}$$

問題

$$\int_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 1}}$$

1. 次の定積分を求めよ。

例題

$$\int_1^e \frac{\log x}{x} dx$$
$$= \int_1^e \log x (\log x)' dx = \left[ \frac{1}{2} (\log x)^2 \right]_1^e$$
$$= \left\{ \frac{1}{2} (\log e)^2 \right\} - \left\{ \frac{1}{2} (\log 1)^2 \right\}$$
$$= \left\{ \frac{1}{2} (2 \log e)^2 \right\} = 2$$

問題

$$\int_1^e \frac{(\log x)^2}{x} dx$$

2. 次の定積分を求めよ。

例題

$$\int_0^1 e^x (e^x - 1)^2 dx$$
$$e^x = t \text{ とおくと}$$
$$\frac{dt}{dx} = e^x, \quad dt = e^x dx$$

$\frac{x}{t}$	$\frac{0}{1}$	$\frac{1}{e}$
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$$\int_0^1 e^x (e^x - 1)^2 dx$$
$$= \int_1^e (t - 1)^2 dt = \left[ \frac{1}{3} (t - 1)^3 \right]_1^e$$
$$= \frac{1}{3} (e - 1)^3$$

問題

$$\int_0^1 e^{-x} (1 - e^{-x})^2 dx$$

例題

$$\int_0^1 x e^{x^2} dx$$
$$= \frac{1}{2} \int_0^1 (x^2)' e^{x^2} dx = \frac{1}{2} \left[ e^{x^2} \right]_0^1$$
$$= \frac{1}{2} (e^1 - e^0) = \frac{1}{2} (e - 1)$$

問題

$$\int_0^1 x^2 e^{x^3} dx$$

例題

$$\int_0^1 \frac{dx}{1 + e^{-x}}$$
$$= \int_0^1 \frac{e^x dx}{(1 + e^{-x}) e^x} = \int_0^1 \frac{e^x dx}{e^x + 1}$$
$$= \int_0^1 \frac{(e^x + 1)'}{(e^x + 1)} dx = \left[ \log(e^x + 1) \right]_0^1$$
$$= \log(e + 1) - \log(1 + 1) = \log\left(\frac{e + 1}{2}\right)$$

問題

$$\int_0^1 \frac{dx}{1 + e^x}$$

例題

$$\int_0^1 e^x (e^x + 1)^3 dx$$
$$= \int_0^1 (e^x + 1)^3 (e^x + 1)' dx$$
$$= \left[ \frac{1}{4} (e^x + 1)^4 \right]_0^1$$
$$= \left\{ \frac{1}{4} (e + 1)^4 \right\} - \left\{ \frac{1}{4} (1 + 1)^4 \right\}$$
$$= \frac{1}{4} (e + 1)^4 - 4$$

問題

$$\int_0^1 e^x (e^x - 1)^4 dx$$

1. 次の定積分を求めよ。

2. 次の定積分を求めよ。

例題

$$\int_1^e \frac{(\log x)^2}{x} dx$$
$$= \int_1^e (\log x)^2 (\log x)' dx = \left[ \frac{1}{3} (\log x)^3 \right]_1^e$$
$$= \left\{ \frac{1}{3} (\log e^3)^3 \right\} - \left\{ \frac{1}{3} (\log 1)^3 \right\}$$
$$= \left\{ \frac{1}{3} (3 \log e)^3 \right\} = 9$$

問題

$$\int_1^e \frac{(\log x)^3}{x} dx$$

例題

$$\int_0^1 x e^{2x^2} dx$$
$$= \frac{1}{4} \int_0^1 (2x^2)' e^{2x^2} dx = \frac{1}{4} \left[ e^{2x^2} \right]_0^1$$
$$= \frac{1}{4} (e^2 - e^0) = \frac{1}{4} (e^2 - 1)$$

問題

$$\int_0^1 x e^{-x^2} dx$$

例題

$$\int_0^1 \frac{e^x}{1 + e^x} dx$$
$$= \int_0^1 \frac{(1 + e^x)'}{(1 + e^x)} dx = \left[ \log (1 + e^x) \right]_0^1$$
$$= \log (e^1 + 1) - \log 2 = \log \left( \frac{e + 1}{2} \right)$$

問題

$$\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

例題

$$\int_0^1 e^x (e^x + 1)^2 dx$$
$$e^x = t \text{ とおくと}$$
$$\frac{dt}{dx} = e^x, \quad dt = e^x dx$$

$\frac{x}{t}$	$\frac{0}{1}$	$\frac{1}{e}$
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$$\int_0^1 e^x (e^x + 1)^2 dx$$
$$= \int_1^e (t + 1)^2 dt = \left[ \frac{1}{3} (t + 1)^3 \right]_1^e$$
$$= \frac{1}{3} (e + 1)^3 - \frac{8}{3}$$

問題

$$\int_0^1 e^{-x} (1 + e^{-x})^2 dx$$

例題

$$\int_0^1 e^x (e^x - 1)^3 dx$$
$$= \int_0^1 (e^x - 1)^3 (e^x - 1)' dx$$
$$= \left[ \frac{1}{4} (e^x - 1)^4 \right]_0^1$$
$$= \left\{ \frac{1}{4} (e - 1)^4 \right\} - \left\{ \frac{1}{4} (1 - 1)^4 \right\}$$
$$= \frac{1}{4} (e - 1)^4$$

問題

$$\int_0^1 e^x (e^x + 1)^4 dx$$



1. 次の定積分を求めよ。

2. 次の定積分を求めよ。

例題

$$\int_2^4 \frac{dx}{x \log x}$$
$$= \int_2^4 \frac{(\log x)'}{(\log x)} dx = \left[ \log |\log x| \right]_2^4$$
$$= \log (\log 4) - \log (\log 2) = \log \left( \frac{2 \log 2}{\log 2} \right)$$
$$= \log 2$$

問題

$$\int_2^4 \frac{dx}{x (\log x)^2}$$

例題

$$\int_0^1 \frac{2 e^x}{e^x + e^{-x}} dx$$
$$= \int_0^1 \frac{2 e^x \times e^x}{(e^x + e^{-x}) \times e^x} dx$$
$$= \int_0^1 \frac{2 e^{2x}}{e^{2x} + 1} dx$$
$$= \int_0^1 \frac{(\frac{1}{2} e^{2x} + 1)'}{(\frac{1}{2} e^{2x} + 1)} dx = \left[ \log \left( \frac{1}{2} e^{2x} + 1 \right) \right]_0^1$$
$$= \log \left( \frac{1}{2} e^2 + 1 \right) - \log 2 = \log \left( \frac{e^2 + 1}{2} \right)$$

問題

$$\int_0^1 \frac{2 e^{-x}}{e^x + e^{-x}} dx$$

例題

$$\int_1^2 \frac{2}{e^x - e^{-x}} dx$$
$$\frac{2}{e^x - e^{-x}} dx = \frac{2 e^x}{e^{2x} - 1} dx$$
$$e^x = t \text{ とおくと } \frac{dt}{dx} = e^x, \quad dt = e^x dx$$
$$\frac{2 e^x}{e^{2x} - 1} dx = \frac{2}{t^2 - 1} dt$$
$$= \left( \frac{1}{t - 1} - \frac{1}{t + 1} \right) dt$$
$$= \log |t - 1| - \log |t + 1| + C$$
$$= \log \left| \frac{t - 1}{t + 1} \right| + C = \log \left| \frac{e^x - 1}{e^x + 1} \right| + C$$
$$\int_1^2 \frac{2}{e^x - e^{-x}} dx = \left[ \log \left| \frac{e^x - 1}{e^x + 1} \right| \right]_1^2$$
$$= \log \left| \frac{e^2 - 1}{e^2 + 1} \right| - \log \left| \frac{e - 1}{e + 1} \right|$$

問題

$$\int_1^2 \frac{2 e^x}{e^x - e^{-x}} dx$$

1. 次の定積分を求めよ。

2. 次の定積分を求めよ。

例題

$$\begin{aligned} & \int_0^{\frac{\pi}{6}} \sin^2 x \cos x \, dx \\ &= \int_0^{\frac{\pi}{6}} \sin^2 x (\sin x)' \, dx \\ &= \left[ -\frac{1}{3} (\sin x)^3 \right]_0^{\frac{\pi}{6}} \\ &= \left\{ -\frac{1}{3} \left( \sin \frac{\pi}{6} \right)^3 \right\} - \left\{ -\frac{1}{3} (\sin 0)^3 \right\} \\ &= -\frac{1}{24} \end{aligned}$$

問題

$$\int_0^{\frac{\pi}{4}} \cos^3 x \sin x \, dx$$

例題

$$\begin{aligned} & \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x + 1} \, dx \\ &= - \int_0^{\frac{\pi}{3}} \frac{(\cos x + 1)'}{(\cos x + 1)} \, dx \\ &= - \left[ \log(\cos x + 1) \right]_0^{\frac{\pi}{3}} \\ &= - \left\{ \log \left( \frac{1}{2} + 1 \right) - \log(1 + 1) \right\} \\ &= \log 2 - \log \frac{3}{2} = \log 2 - \log 3 + \log 2 \\ &= \log \frac{4}{3} \end{aligned}$$

問題

$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{\sin x + 1} \, dx$$

例題

$$\begin{aligned} & \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\sin x} \, dx \\ &= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{\sin^2 x} \, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1 - \cos^2 x} \, dx \\ t = \cos x \text{ とおくと } \frac{dt}{dx} &= -\sin x \quad \left. x \right|_{\frac{\pi}{3}}^{\frac{\pi}{2}} \quad \left. \frac{1}{3} \right|_{\frac{1}{2}}^0 \\ \sin x \, dx &= -dt \quad \left. t \right|_{\frac{1}{2}}^0 \\ \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1 - \cos^2 x} \, dx &= \int_{\frac{1}{2}}^0 \frac{-1}{1 - t^2} \, dt \\ &= \int_0^{\frac{1}{2}} \frac{1}{1 - t^2} \, dt \\ &= \frac{1}{2} \int_0^{\frac{1}{2}} \left( \frac{1}{1 + t} + \frac{1}{1 - t} \right) \, dt \\ &= \frac{1}{2} \left[ \log |1 + t| - \log |1 - t| \right]_0^{\frac{1}{2}} \\ &= \frac{1}{2} \left\{ \log \left| 1 + \frac{1}{2} \right| - \log \left| 1 - \frac{1}{2} \right| \right\} \\ &\quad - \frac{1}{2} \left\{ \log |1 + 0| - \log |1 - 0| \right\} \\ &= \frac{1}{2} \left\{ \log \frac{3}{2} - \log \frac{1}{2} \right\} = \frac{1}{2} \log 3 \end{aligned}$$

問題

$$\int_0^{\frac{\pi}{6}} \frac{1}{\cos x} \, dx$$

1. 次の三角関数を微分せよ。

例題

$$\frac{1}{\tan x} = \frac{\cos x}{\sin x}$$
$$\left(\frac{1}{\tan x}\right)' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{\sin^2 x}$$
$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

問題

$$\tan x = \frac{\sin x}{\cos x}$$

2. 次の定積分を求めよ。

例題

$$\frac{3}{4} \frac{1}{\sin^2 x} dx$$
$$= \left[-\frac{1}{\tan x}\right]_{\frac{3}{4}}^{\frac{3}{4}} = \left(-\frac{1}{\tan \frac{3}{4}}\right) - \left(-\frac{1}{\tan \frac{3}{4}}\right)$$
$$= 1 - \frac{1}{\sqrt{3}}$$

問題

$$\frac{3}{4} \frac{1}{\cos^2 x} dx$$

例題

$$\frac{2}{4} \frac{1}{\tan x} dx$$
$$= \frac{2}{4} \frac{\cos x}{\sin x} dx = \frac{2}{4} \frac{(\sin x)'}{\sin x} dx$$
$$= \left[\log \left|\sin x\right|\right]_{\frac{2}{4}}^{\frac{2}{4}} = \log \left|1\right| - \log \left|\frac{\sqrt{2}}{2}\right|$$
$$= -\log \frac{\sqrt{2}}{2} = -\frac{1}{2} \log 2$$

問題

$$\frac{3}{0} \tan x dx$$

3. 次の定積分を求めよ。

例題

$$\frac{3}{0} \cos^2 x \sin x dx$$
$$= -\frac{3}{0} \cos^2 x (\cos x)'$$
$$= -\left[\frac{1}{3} (\cos x)^3\right]_{\frac{3}{0}}^{\frac{3}{0}}$$
$$= -\left\{\frac{1}{3} (\cos \frac{3}{3})^3\right\} + \left\{\frac{1}{3} (\cos 0)^3\right\}$$
$$= -\frac{1}{3} \left(\frac{1}{2}\right)^3 + \frac{1}{3} (1)^3 = \frac{7}{24}$$

問題

$$\frac{4}{0} \sin^3 x \cos x dx$$

例題

$$\frac{2}{0} \sin^3 x dx$$
$$= \frac{2}{0} (1 - \cos^2 x) \sin x dx$$
$$\cos x = t \text{ とおくと } \frac{dt}{dx} = -\sin x$$
$$\frac{2}{0} (1 - \cos^2 x) \sin x dx \quad \begin{array}{c|c} x & 0 \\ \hline t & 1 \end{array} \quad \begin{array}{c|c} 2 & 0 \\ \hline & 0 \end{array}$$
$$= -\int_1^0 (1 - t^2) dt = \int_0^1 (1 - t^2) dt$$
$$= \left[t - \frac{t^3}{3}\right]_0^1 = \left(1 - \frac{1^3}{3}\right) - \left(0 - \frac{0^3}{3}\right) = \frac{2}{3}$$

問題

$$\frac{2}{0} \cos^3 x dx$$