

1. 次の定積分を求めよ。

Find the following definite integral.
2. 次の定積分を求めよ。

Find the following definite integral.

れいだい  
例題

$$\int_0^2 x(2-x)^2 dx$$
$$t=2-x \text{ とおくとき } \frac{dt}{dx}=-1, \quad dx=(-1) dt$$
$$\int_0^2 x(2-x)^2 dx \qquad \frac{x}{t} \parallel \begin{array}{l} 0 \rightarrow 2 \\ 2 \rightarrow 0 \end{array}$$
$$= \int_2^0 (2-t)t^2(-1) dt = \int_2^0 (t^3-2t^2) dt$$
$$= \left[ \frac{t^4}{4} - \frac{2t^3}{3} \right]_2^0$$
$$= \left( \frac{0^4}{4} - \frac{2 \times 0^3}{3} \right) - \left( \frac{2^4}{4} - \frac{2 \times 2^3}{3} \right) = -\frac{4}{3}$$

もんだい  
問題①

$$\int_0^3 x(3-x)^2 dx$$

もんだい  
問題②

$$\int_0^1 x(x-1)^2 dx$$

れいだい  
例題

$$\int_0^3 x\sqrt{x+1} dx$$
$$t=\sqrt{x+1} \text{ とおくとき } x=t^2-1$$
$$\frac{dx}{dt}=2t, \quad dx=2t dt \qquad \frac{x}{t} \parallel \begin{array}{l} 0 \rightarrow 2 \\ 2 \rightarrow 0 \end{array}$$
$$\int_0^3 x\sqrt{x+1} dx$$
$$= \int_1^2 (t^2-1)t \times 2t dt = \int_1^2 (2t^4-2t^2) dt$$
$$= \left[ \frac{2}{5}t^5 - \frac{2}{3}t^3 \right]_1^2$$
$$= \left( \frac{64}{5} - \frac{16}{3} \right) - \left( \frac{2}{5} - \frac{2}{3} \right) = \frac{116}{15}$$

もんだい  
問題①

$$\int_2^3 x\sqrt{x-2} dx$$

もんだい  
問題②

$$\int_0^1 x\sqrt{1-x} dx$$

1. 次の定積分を計算せよ。Find the following definite integral.

2. 次の定積分を計算せよ。Find the following definite integral.

れいだい  
例題

$$\int_0^1 x(1-x)^3 dx$$
$$t=1-x \text{ とおくと } \frac{dt}{dx}=-1, \quad dx=(-1) dt$$
$$\int_0^1 x(1-x)^3 dx \qquad \left. \begin{matrix} x \\ t \end{matrix} \right\| \begin{matrix} 0 \rightarrow 1 \\ 1 \rightarrow 0 \end{matrix}$$
$$= \int_1^0 (1-t)t^3 \times (-1) dt = \int_1^0 (t^4 - t^3) dt$$
$$= \left[ \frac{1}{5}t^5 - \frac{1}{4}t^4 \right]_1^0$$
$$= \left( \frac{0}{5} - \frac{0}{4} \right) - \left( \frac{1}{5} - \frac{1}{4} \right) = -\frac{1}{20}$$

もんだい  
問題①

$$\int_0^2 x(2-x)^4 dx$$

もんだい  
問題②

$$\int_0^1 x(1-x)^5 dx$$

れいだい  
例題①

$$\int_0^5 \frac{1}{\sqrt{3x+1}} dx$$
$$t=\sqrt{3x+1} \text{ とおくと } x=\frac{t^2-1}{3}$$
$$\frac{dx}{dt}=\frac{2}{3}t, \quad dx=\frac{2}{3}t dt \qquad \left. \begin{matrix} x \\ t \end{matrix} \right\| \begin{matrix} 0 \rightarrow 5 \\ 1 \rightarrow 4 \end{matrix}$$
$$\int_0^5 \frac{1}{\sqrt{3x+1}} dx = \frac{2}{3} \int_1^4 \frac{1}{t} \times t dt$$
$$= \frac{2}{3} \left[ t \right]_1^4 = \frac{2}{3}(4-1) = 2$$

もんだい  
問題①

$$\int_0^4 \frac{1}{\sqrt{2x+1}} dx$$

れいだい  
例題②

$$\int_0^3 \sqrt{4-x} dx$$
$$t=\sqrt{4-x} \text{ とおくと } x=4-t^2$$
$$\frac{dx}{dt}=-2t, \quad dx=-2t dt \qquad \left. \begin{matrix} x \\ t \end{matrix} \right\| \begin{matrix} 0 \rightarrow 3 \\ 2 \rightarrow 1 \end{matrix}$$
$$\int_0^3 \sqrt{4-x} dx = \int_2^1 t \times (-2t) dt$$
$$= -2 \int_2^1 t^2 dt = -2 \left[ \frac{t^3}{3} \right]_2^1$$
$$= -2 \left( \frac{1}{3} - \frac{8}{3} \right) = \frac{14}{3}$$

もんだい  
問題②

$$\int_1^5 \sqrt{5-x} dx$$

1. 次の定積分を計算せよ。

Find the following definite integral.
2. 次の定積分を計算せよ。

Find the following definite integral.

れいだい

例題①

$$\int_0^1 (2x + 1)^3 dx$$

$$t = 2x + 1 \quad \text{とおくと}$$

$$\frac{dt}{dx} = 2, \quad dx = \frac{1}{2} dt \qquad \frac{x}{t} \parallel \begin{matrix} 0 \rightarrow 1 \\ 1 \rightarrow 3 \end{matrix}$$

$$\int_0^1 (2x + 1)^3 dx = \int_1^3 t^3 \times \frac{1}{2} dt$$

$$= \left[ \frac{t^4}{4} \right]_1^3 = \frac{81}{4} - \frac{1}{4} = 20$$

れいだい

例題①

$$\int_0^2 (3x - 1)^3 dx$$

れいだい

例題②

$$\int_0^1 \frac{2x}{x^2 + 1} dx$$

$$\int_0^1 \frac{2x}{x^2 + 1} dx = \int_0^1 \frac{(x^2 + 1)'}{x^2 + 1} dx$$

$$= \left[ \log(x^2 + 1) \right]_0^1 = \log 2 - \log 1 = \log 2$$

もんだい

問題②

$$\int_0^2 \frac{2x}{x^2 + 4} dx$$

もんだい

問題②

$$\int_0^1 \frac{3x^2}{x^3 + 1} dx$$

れいだい

例題①

$$\int_0^1 \frac{x dx}{\sqrt{x^2 + 1}}$$

$$t = \sqrt{x^2 + 1} \quad \text{とおくと}$$

$$\frac{dt}{dx} = \frac{x}{\sqrt{x^2 + 1}}, \quad dt = \frac{x dx}{\sqrt{x^2 + 1}} \quad \frac{x}{t} \parallel \begin{matrix} 0 \rightarrow 1 \\ 1 \rightarrow \sqrt{2} \end{matrix}$$

$$\int_0^1 \frac{x dx}{\sqrt{x^2 + 1}} = \int_1^{\sqrt{2}} dt = \left[ t \right]_1^{\sqrt{2}} = \sqrt{2} - 1$$

もんだい

問題①

$$\int_0^4 \frac{x dx}{\sqrt{x^2 + 4}}$$

れいだい

例題②

$$\int_0^1 x \sqrt{x^2 + 3} dx$$

$$t = \sqrt{x^2 + 3} \quad \text{とおくと}, \quad x^2 = t^2 - 3$$

$$2x \frac{dx}{dt} = 2t, \quad x dx = t dt \qquad \frac{x}{t} \parallel \begin{matrix} 0 \rightarrow 1 \\ \sqrt{3} \rightarrow 2 \end{matrix}$$

$$x \rightarrow 0 \quad \text{のとき} \quad t \rightarrow \sqrt{3}, \quad x \rightarrow 1 \quad \text{のとき} \quad t \rightarrow 2$$

$$\int_0^1 x \sqrt{x^2 + 1} dx = \int_{\sqrt{3}}^2 t \times t dt$$

$$= \int_{\sqrt{3}}^2 t^2 dt = \left[ \frac{t^3}{3} \right]_{\sqrt{3}}^2 = \frac{8}{3} - \sqrt{3}$$

もんだい

問題②

$$\int_0^1 x \sqrt{x^2 + 1} dx$$

1. 次の定積分を計算せよ。Find the following definite integral.

2. 次の定積分を計算せよ。Find the following definite integral.

例題

$$\int_0^3 \sqrt{9-x^2} \, dx$$

$$x=3 \sin \theta \text{ とおくと}$$

$$dx=3 \cos \theta \, d\theta$$

$$\frac{x}{\theta} \parallel \begin{matrix} 0 \rightarrow 3 \\ 0 \rightarrow \frac{\pi}{2} \end{matrix}$$

この範囲では  $\cos \theta \geq 0$  である。

$$\sqrt{9-x^2} = \sqrt{9-(3 \sin \theta)^2} = \sqrt{9(1-\sin^2 \theta)}$$

$$= \sqrt{9 \cos^2 \theta} = 3 \cos \theta$$

$$\int_0^3 \sqrt{9-x^2} \, dx = \int_0^{\frac{\pi}{2}} (3 \cos \theta) \times 3 \cos \theta \, d\theta$$

$$= 9 \int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta = 9 \int_0^{\frac{\pi}{2}} \left( \frac{1+\cos 2\theta}{2} \right) d\theta$$

$$= \frac{9}{2} \left[ \theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{2}}$$

$$= \frac{9}{2} \left\{ \left( \frac{\pi}{2} + \frac{1}{2} \sin \pi \right) - \left( 0 + \frac{1}{2} \sin 0 \right) \right\}$$

$$= \frac{9}{4} \pi$$

問題①

$$\int_{-1}^1 \sqrt{1-x^2} \, dx$$

問題②

$$\int_0^1 \sqrt{4-x^2} \, dx$$

例題

$$\int_0^3 \frac{dx}{x^2+9}$$

$$x=3 \tan \theta \text{ とおくと}$$

$$dx = \frac{3 \, d\theta}{\cos^2 \theta}$$

$$\frac{x}{\theta} \parallel \begin{matrix} 0 \rightarrow 3 \\ 0 \rightarrow \frac{\pi}{4} \end{matrix}$$

$$\int_0^3 \frac{dx}{x^2+9} = \int_0^{\frac{\pi}{4}} \frac{1}{(3 \tan \theta)^2+9} \times \frac{3 \, d\theta}{\cos^2 \theta}$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{9(\tan^2 \theta+1)} \times \frac{3 \, d\theta}{\cos^2 \theta}$$

$$= \int_0^{\frac{\pi}{4}} \frac{\cos^2 \theta}{9} \times \frac{3 \, d\theta}{\cos^2 \theta} = \frac{1}{3} \int_0^{\frac{\pi}{4}} d\theta$$

$$= \frac{1}{3} \left[ \theta \right]_0^{\frac{\pi}{4}} = \frac{1}{3} \left( \frac{\pi}{4} - 0 \right)$$

$$= \frac{\pi}{12}$$

問題①

$$\int_0^1 \frac{dx}{x^2+1}$$

問題②

$$\int_{-2}^2 \frac{dx}{x^2+4}$$

1. 次の定積分を計算せよ。

Find the following definite integral.
2. 次の定積分を計算せよ。

Find the following definite integral.

れいだい例題

$$\int_0^3 \frac{1}{\sqrt{9-x^2}} dx$$

$x=3 \sin \theta$  とおくと

$dx=3 \cos \theta d\theta$

$\frac{x}{\theta} \parallel \begin{matrix} 0 \rightarrow 3 \\ 0 \rightarrow \frac{\pi}{2} \end{matrix}$

この範囲では  $\cos \theta \geq 0$  である。

$$\begin{aligned} \sqrt{9-x^2} &= \sqrt{9-(3 \sin \theta)^2} = \sqrt{9(1-\sin^2 \theta)} \\ &= \sqrt{9 \cos^2 \theta} = 3 \cos \theta \end{aligned}$$

$$\begin{aligned} &\int_0^3 \frac{1}{\sqrt{9-x^2}} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{1}{3 \cos \theta} \times 3 \cos \theta d\theta \\ &= \int_0^{\frac{\pi}{2}} d\theta = \left[ \theta \right]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 0 = \frac{\pi}{2} \end{aligned}$$

もんだい問題

①

$$\int_0^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}} dx$$

もんだい問題

②

$$\int_0^2 \frac{1}{\sqrt{16-x^2}} dx$$

れいだい例題

$$\int_0^{\sqrt{3}} \frac{dx}{(x^2+3)^2}$$

$x=\sqrt{3} \tan \theta$  とおくと

$dx = \frac{\sqrt{3} d\theta}{\cos^2 \theta}$

$\frac{x}{\theta} \parallel \begin{matrix} 0 \rightarrow \sqrt{3} \\ 0 \rightarrow \frac{\pi}{4} \end{matrix}$

$$\begin{aligned} &\int_0^{\sqrt{3}} \frac{dx}{(x^2+3)^2} \\ &= \int_0^{\frac{\pi}{4}} \frac{1}{(3 \tan^2 \theta + 3)^2} \times \frac{\sqrt{3} d\theta}{\cos^2 \theta} \\ &= \int_0^{\frac{\pi}{4}} \frac{1}{9 (\tan^2 \theta + 1)^2} \times \frac{\sqrt{3} d\theta}{\cos^2 \theta} \\ &= \int_0^{\frac{\pi}{4}} \frac{\cos^4 \theta}{9} \times \frac{\sqrt{3} d\theta}{\cos^2 \theta} \\ &= \frac{\sqrt{3}}{9} \int_0^{\frac{\pi}{4}} \cos^2 \theta d\theta \\ &= \frac{\sqrt{3}}{9} \int_0^{\frac{\pi}{4}} \frac{1+\cos 2\theta}{2} d\theta \\ &= \frac{\sqrt{3}}{18} \left[ \theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{4}} = \frac{\sqrt{3}}{72} (\pi + 2) \\ &\quad - \frac{\sqrt{3}}{18} \left\{ \left( \frac{\pi}{4} + \frac{1}{2} \sin \frac{\pi}{2} \right) - \left( 0 + \frac{1}{2} \sin 0 \right) \right\} \end{aligned}$$

もんだい問題

$$\int_0^1 \frac{dx}{(x^2+1)^2}$$

1. 次の定積分を計算せよ。

Find the following definite integral.
2. 次の定積分を計算せよ。

Find the following definite integral.

れいだい例題

$$\int_0^2 x^2 \sqrt{4 - x^2} \, dx$$

$x = 2 \sin \theta$  とおくと

$$\frac{x}{\theta} \parallel \begin{array}{l} 0 \rightarrow 2 \\ 0 \rightarrow \frac{\pi}{2} \end{array}$$

$dx = 2 \cos \theta \, d\theta$

はんいはんいこの範囲では  $\cos \theta \geq 0$  である。

$$\begin{aligned} \sqrt{4 - x^2} &= \sqrt{4 - (2 \sin \theta)^2} = \sqrt{4 (1 - \sin^2 \theta)} \\ &= \sqrt{4 \cos^2 \theta} = 2 \cos \theta \end{aligned}$$

$$\begin{aligned} \int_0^2 x^2 \sqrt{4 - x^2} \, dx &= \int_0^{\frac{\pi}{2}} (2 \sin \theta)^2 \times 2 \cos \theta \times 2 \cos \theta \, d\theta \\ &= 16 \int_0^{\frac{\pi}{2}} \sin^2 \theta \cos^2 \theta \, d\theta \\ &= 16 \int_0^{\frac{\pi}{2}} \left( \frac{1}{2} \sin 2\theta \right)^2 \, d\theta \\ &= 4 \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2\theta}{2} \, d\theta \\ &= 2 \left[ \theta - \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{2}} \\ &= 2 \left\{ \left( \frac{\pi}{2} - \frac{1}{2} \sin \pi \right) - \left( 0 - \frac{1}{2} \sin 0 \right) \right\} = \pi \end{aligned}$$

もんだい問題

$$\int_0^1 x^2 \sqrt{1 - x^2} \, dx$$

れいだい例題

$$\int_0^1 \frac{dx}{\sqrt{x^2 + 3}}$$

$x = \sqrt{3} \tan \theta$  とおくと

$$\frac{x}{\theta} \parallel \begin{array}{l} 0 \rightarrow 1 \\ 0 \rightarrow \frac{\pi}{6} \end{array}$$

$dx = \frac{\sqrt{3} \, d\theta}{\cos^2 \theta}$

$$\frac{1}{\sqrt{x^2 + 3}} = \frac{1}{\sqrt{3 (1 + \tan^2 \theta)}} = \frac{\cos \theta}{\sqrt{3}}$$

$$\begin{aligned} \int_0^1 \frac{dx}{\sqrt{x^2 + 3}} &= \int_0^{\frac{\pi}{6}} \frac{\cos \theta}{\sqrt{3}} \times \frac{\sqrt{3} \, d\theta}{\cos^2 \theta} \\ &= \int_0^{\frac{\pi}{6}} \frac{d\theta}{\cos \theta} = \frac{1}{2} \log 3 \end{aligned}$$

もんだい問題①

$$\int_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 3}}$$

もんだい問題②

$$\int_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 1}}$$

1. 次の定積分を求めよ。Find the following definite integral.

2. 次の定積分を求めよ。Find the following definite integral.

れいだい  
例題  
①

$$\int_1^{e^2} \frac{\log x}{x} dx$$
$$= \int_1^{e^2} \log x (\log x)' dx = \left[ \frac{1}{2} (\log x)^2 \right]_1^{e^2}$$
$$= \left\{ \frac{1}{2} (\log e^2)^2 \right\} - \left\{ \frac{1}{2} (\log 1)^2 \right\}$$
$$= \left\{ \frac{1}{2} (2 \log e)^2 \right\} = 2$$

もんだい  
問題  
①

$$\int_1^{e^2} \frac{(\log x)^2}{x} dx$$

れいだい  
例題  
②

$$\int_0^1 x e^{x^2} dx$$
$$= \frac{1}{2} \int_0^1 (x^2)' e^{x^2} dx = \frac{1}{2} \left[ e^{x^2} \right]_0^1$$
$$= \frac{1}{2} (e^1 - e^0) = \frac{1}{2} (e - 1)$$

もんだい  
問題  
②

$$\int_0^1 x^2 e^{x^3} dx$$

れいだい  
例題  
③

$$\int_0^1 \frac{dx}{1 + e^{-x}}$$
$$= \int_0^1 \frac{e^x dx}{(1 + e^{-x}) e^x} = \int_0^1 \frac{e^x dx}{e^x + 1}$$
$$= \int_0^1 \frac{(e^x + 1)'}{(e^x + 1)} dx = \left[ \log (e^x + 1) \right]_0^1$$
$$= \log (e + 1) - \log (1 + 1) = \log \left( \frac{e + 1}{2} \right)$$

もんだい  
問題  
③

$$\int_0^1 \frac{dx}{1 + e^x}$$

れいだい  
例題  
①

$$\int_0^1 e^x (e^x - 1)^2 dx$$

$e^x = t$  とおくと

$$\frac{dt}{dx} = e^x, \quad dt = e^x dx$$

|     |                   |
|-----|-------------------|
| $x$ | $0 \rightarrow 1$ |
| $t$ | $1 \rightarrow e$ |

$$\int_0^1 e^x (e^x - 1)^2 dx$$
$$= \int_1^e (t - 1)^2 dt = \left[ \frac{1}{3} (t - 1)^3 \right]_1^e$$
$$= \frac{1}{3} (e - 1)^3$$

もんだい  
問題  
①

$$\int_0^1 e^{-x} (1 - e^{-x})^2 dx$$

れいだい  
例題  
②

$$\int_0^1 e^x (e^x + 1)^3 dx$$
$$= \int_0^1 (e^x + 1)^3 (e^x + 1)' dx$$
$$= \left[ \frac{1}{4} (e^x + 1)^4 \right]_0^1$$
$$= \left\{ \frac{1}{4} (e + 1)^4 \right\} - \left\{ \frac{1}{4} (1 + 1)^4 \right\}$$
$$= \frac{1}{4} (e + 1)^4 - 4$$

もんだい  
問題  
②

$$\int_0^1 e^x (e^x - 1)^4 dx$$

1. 次の定積分を求めよ。Find the following definite integral.

2. 次の定積分を求めよ。Find the following definite integral.

れいだい  
例題  
①

$$\int_1^{e^3} \frac{(\log x)^2}{x} dx$$
$$= \int_1^{e^3} (\log x)^2 (\log x)' dx = \left[ \frac{1}{3} (\log x)^3 \right]_1^{e^3}$$
$$= \left\{ \frac{1}{3} (\log e^3)^3 \right\} - \left\{ \frac{1}{3} (\log 1)^3 \right\}$$
$$= \left\{ \frac{1}{3} (3 \log e)^3 \right\} = 9$$

もんだい  
問題  
①

$$\int_1^{e^4} \frac{(\log x)^3}{x} dx$$

れいだい  
例題  
②

$$\int_0^1 x e^{2x^2} dx$$
$$= \frac{1}{4} \int_0^1 (2x^2)' e^{2x^2} dx = \frac{1}{4} \left[ e^{2x^2} \right]_0^1$$
$$= \frac{1}{4} (e^2 - e^0) = \frac{1}{4} (e^2 - 1)$$

もんだい  
問題  
②

$$\int_0^1 x e^{-x^2} dx$$

れいだい  
例題  
③

$$\int_0^1 \frac{e^x}{1 + e^x} dx$$
$$= \int_0^1 \frac{(1 + e^x)'}{(1 + e^x)} dx = \left[ \log (1 + e^x) \right]_0^1$$
$$= \log (e^1 + 1) - \log 2 = \log \left( \frac{e + 1}{2} \right)$$

もんだい  
問題  
③

$$\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

れいだい  
例題  
①

$$e^x = t \quad \text{とおくと}$$
$$\frac{dt}{dx} = e^x, \quad dt = e^x dx$$

|     |                   |
|-----|-------------------|
| $x$ | $0 \rightarrow 1$ |
| $t$ | $1 \rightarrow e$ |

$$\int_0^1 e^x (e^x + 1)^2 dx$$
$$= \int_1^e (t + 1)^2 dt = \left[ \frac{1}{3} (t + 1)^3 \right]_1^e$$
$$= \frac{1}{3} (e + 1)^3 - \frac{8}{3}$$

もんだい  
問題  
①

$$\int_0^1 e^{-x} (1 + e^{-x})^2 dx$$

れいだい  
例題  
②

$$\int_0^1 e^x (e^x - 1)^3 dx$$
$$= \int_0^1 (e^x - 1)^3 (e^x - 1)' dx$$
$$= \left[ \frac{1}{4} (e^x - 1)^4 \right]_0^1$$
$$= \left\{ \frac{1}{4} (e - 1)^4 \right\} - \left\{ \frac{1}{4} (1 - 1)^4 \right\}$$
$$= \frac{1}{4} (e - 1)^4$$

もんだい  
問題  
②

$$\int_0^1 e^x (e^x + 1)^4 dx$$



1. 次の定積分を求めよ。

つぎていせきぶんもと

Find the following definite integral.

2. 次の定積分を求めよ。

つぎていせきぶんもと

Find the following definite integral.

れいだい  
例題  
①

$$\int_2^4 \frac{dx}{x \log x}$$
$$= \int_2^4 \frac{(\log x)'}{(\log x)} dx = \left[ \log |\log x| \right]_2^4$$
$$= \log (\log 4) - \log (\log 2) = \log \left( \frac{2 \log 2}{\log 2} \right)$$
$$= \log 2$$

もんだい  
問題  
①

$$\int_2^4 \frac{dx}{x (\log x)^2}$$

れいだい  
例題  
②

$$\int_0^1 \frac{2 e^x}{e^x + e^{-x}} dx$$
$$= \int_0^1 \frac{2 e^x \times e^x}{(e^x + e^{-x}) \times e^x} dx$$
$$= \int_0^1 \frac{2 e^{2x}}{e^{2x} + 1} dx$$
$$= \int_0^1 \frac{(e^{2x} + 1)'}{(e^{2x} + 1)} dx = \left[ \log (e^{2x} + 1) \right]_0^1$$
$$= \log (e^2 + 1) - \log 2 = \log \left( \frac{e^2 + 1}{2} \right)$$

もんだい  
問題  
②

$$\int_0^1 \frac{2 e^{-x}}{e^x + e^{-x}} dx$$

れいだい  
例題

$$\int_1^2 \frac{2}{e^x - e^{-x}} dx$$
$$\int \frac{2}{e^x - e^{-x}} dx = \int \frac{2 e^x}{e^{2x} - 1} dx$$
$$e^x = t \text{ とおくと } \frac{dt}{dx} = e^x, \quad dt = e^x dx$$
$$\int \frac{2 e^x}{e^{2x} - 1} dx = \int \frac{2}{t^2 - 1} dt$$
$$= \int \left( \frac{1}{t - 1} - \frac{1}{t + 1} \right) dt$$
$$= \log |t - 1| - \log |t + 1| + C$$
$$= \log \left| \frac{t - 1}{t + 1} \right| + C = \log \left| \frac{e^x - 1}{e^x + 1} \right| + C$$
$$\int_1^2 \frac{2}{e^x - e^{-x}} dx = \left[ \log \left| \frac{e^x - 1}{e^x + 1} \right| \right]_1^2$$
$$= \log \left| \frac{e^2 - 1}{e^2 + 1} \right| - \log \left| \frac{e - 1}{e + 1} \right|$$

もんだい  
問題

$$\int_1^2 \frac{2 e^x}{e^x - e^{-x}} dx$$

1. 次の定積分を求めよ。

つぎていせきぶんもと

Find the following definite integral.

2. 次の定積分を求めよ。

つぎていせきぶんもと

Find the following definite integral.

れいだい  
例題  
①

$$\int_0^{\frac{\pi}{6}} \sin^2 x \cos x \, dx$$
$$= \int_0^{\frac{\pi}{6}} \sin^2 x (\sin x)' \, dx$$
$$= \left[ \frac{1}{3} (\sin x)^3 \right]_0^{\frac{\pi}{6}}$$
$$= \left\{ \frac{1}{3} \left( \sin \frac{\pi}{6} \right)^3 \right\} - \left\{ \frac{1}{3} (\sin 0)^3 \right\}$$
$$= \frac{1}{24}$$

もんだい  
問題  
①

$$\int_0^{\frac{\pi}{4}} \cos^3 x \sin x \, dx$$

れいだい  
例題  
②

$$\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x + 1} \, dx$$
$$= - \int_0^{\frac{\pi}{3}} \frac{(\cos x + 1)'}{(\cos x + 1)} \, dx$$
$$= - \left[ \log (\cos x + 1) \right]_0^{\frac{\pi}{3}}$$
$$= - \left\{ \log \left( \frac{1}{2} + 1 \right) - \log (1 + 1) \right\}$$
$$= \log 2 - \log \frac{3}{2} = \log 2 - \log 3 + \log 2$$
$$= \log \frac{4}{3}$$

もんだい  
問題  
②

$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{\sin x + 1} \, dx$$

れいだい  
例題

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\sin x} \, dx$$
$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{\sin^2 x} \, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1 - \cos^2 x} \, dx$$

$t = \cos x$ とおくと $\frac{dt}{dx} = -\sin x$

$x \parallel \frac{\pi}{3} \rightarrow \frac{\pi}{2}$

$\sin x \, dx = -dt$

$t \parallel \frac{1}{2} \rightarrow 0$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1 - \cos^2 x} \, dx = \int_{\frac{1}{2}}^0 \frac{-1}{1 - t^2} \, dt$$
$$= \int_0^{\frac{1}{2}} \frac{1}{1 - t^2} \, dt$$
$$= \frac{1}{2} \int_0^{\frac{1}{2}} \left( \frac{1}{1 + t} + \frac{1}{1 - t} \right) \, dt$$
$$= \frac{1}{2} \left[ \log |1 + t| - \log |1 - t| \right]_0^{\frac{1}{2}}$$
$$= -\frac{1}{2} \left\{ \log \left| 1 + \frac{1}{2} \right| - \log \left| 1 - \frac{1}{2} \right| \right\}$$
$$= -\frac{1}{2} \left\{ \log \left| 1 + 0 \right| - \log \left| 1 - 0 \right| \right\}$$
$$= -\frac{1}{2} \left\{ \log \frac{3}{2} - \log \frac{1}{2} \right\} = \frac{1}{2} \log 3$$

もんだい  
問題

$$\int_0^{\frac{\pi}{6}} \frac{1}{\cos x} \, dx$$

1. 次の三角関数を微分せよ。

Differentiate the following trigonometric functions.

3. 次の定積分を求めよ。

Find the following definite integral.

例題

$$\frac{1}{\tan x} = \frac{\cos x}{\sin x}$$
$$\left(\frac{1}{\tan x}\right)' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{\sin^2 x}$$
$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

問題

$$\tan x = \frac{\sin x}{\cos x}$$

2. 次の定積分を求めよ。

Find the following definite integral.

例題

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sin^2 x} dx$$
$$= \left[-\frac{1}{\tan x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left(-\frac{1}{\tan \frac{\pi}{3}}\right) - \left(-\frac{1}{\tan \frac{\pi}{4}}\right)$$
$$= 1 - \frac{1}{\sqrt{3}}$$

問題

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x} dx$$

例題

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\tan x} dx$$
$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(\sin x)'}{\sin x} dx$$
$$= \left[\log \left|\sin x\right|\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \log \left|1\right| - \log \left|\frac{\sqrt{2}}{2}\right|$$
$$= -\log \frac{\sqrt{2}}{2} = \frac{1}{2} \log 2$$

問題

$$\int_0^{\frac{\pi}{3}} \tan x dx$$

例題

$$\int_0^{\frac{\pi}{3}} \cos^2 x \sin x dx$$
$$= -\int_0^{\frac{\pi}{3}} \cos^2 x (\cos x)' dx$$
$$= -\left[\frac{1}{3}(\cos x)^3\right]_0^{\frac{\pi}{3}}$$
$$= -\left\{\frac{1}{3}\left(\cos \frac{\pi}{3}\right)^3\right\} + \left\{\frac{1}{3}(\cos 0)^3\right\}$$
$$= -\frac{1}{3}\left(\frac{1}{2}\right)^3 + \frac{1}{3}(1)^3 = \frac{7}{24}$$

問題

$$\int_0^{\frac{\pi}{4}} \sin^3 x \cos x dx$$

例題

$$\int_0^{\frac{\pi}{2}} \sin^3 x dx$$
$$= \int_0^{\frac{\pi}{2}} (1 - \cos^2 x) \sin x dx$$
$$\cos x = t \quad \text{とおくと} \quad \frac{dt}{dx} = -\sin x$$
$$\int_0^{\frac{\pi}{2}} (1 - \cos^2 x) \sin x dx \qquad \frac{x}{t} \parallel \begin{array}{l} 0 \rightarrow \frac{\pi}{2} \\ 1 \rightarrow 0 \end{array}$$
$$= -\int_1^0 (1 - t^2) dt = \int_0^1 (1 - t^2) dt$$
$$= \left[t - \frac{t^3}{3}\right]_0^1 = \left(1 - \frac{1^3}{3}\right) - \left(0 - \frac{0^3}{3}\right) = \frac{2}{3}$$

問題

$$\int_0^{\frac{\pi}{2}} \cos^3 x dx$$