

1. 次の不定積分を求めよ。 Find the following indefinite integral.

$$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$$

例題①

$$\begin{aligned} \int x \sin x \, dx &= \int x (-\cos x)' \, dx \\ &= x (-\cos x) - \int (x)' (-\cos x) \, dx \\ &= -x \cos x + \sin x + C \end{aligned}$$

問題①

$$\int x \cos x \, dx$$

例題②

$$\begin{aligned} \int x^2 \cos x \, dx &= \int x^2 (\sin x)' \, dx \\ &= x^2 \sin x - \int (x^2)' \sin x \, dx \\ &= x^2 \sin x - 2 \int x \sin x \, dx \\ &= x^2 \sin x + 2 x \cos x - 2 \sin x + C \end{aligned}$$

問題②

$$\int x^2 \sin x \, dx$$

例題③

$$\begin{aligned} \int \log(x+1) \, dx &= \int \log(x+1) \times (x+1)' \, dx \\ &= (x+1) \log(x+1) - \int \frac{x+1}{x+1} \, dx \\ &= (x+1) \log(x+1) - \int 1 \, dx \\ &= (x+1) \log(x+1) - x + C \end{aligned}$$

問題③

$$\int \log(x+2) \, dx$$

2. 次の不定積分を求めよ。 Find the following indefinite integral.

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}, \quad \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\sin \alpha \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

例題①

$$\begin{aligned} \int \cos^2 x \, dx \\ &= \int \frac{1 + \cos 2x}{2} \, dx = \frac{1}{2} x + \frac{1}{4} \sin 2x + C \end{aligned}$$

問題①

$$\int \sin^2 x \, dx$$

例題②

$$\begin{aligned} \int \sin^2 3x \, dx \\ &= \int \frac{1 - \cos 6x}{2} \, dx = \frac{1}{2} x - \frac{1}{12} \sin 6x + C \end{aligned}$$

問題②

$$\int \cos^2 4x \, dx$$

例題③

$$\begin{aligned} \int \sin 2x \cos x \, dx \\ &= \int \frac{1}{2} \{ \sin(2x+x) + \sin(2x-x) \} \, dx \\ &= \frac{1}{2} \int (\sin 3x + \sin x) \, dx \\ &= \frac{1}{2} \left( -\frac{1}{3} \cos 3x - \cos x \right) + C \\ &= -\frac{1}{6} \cos 3x - \frac{1}{2} \cos x + C \end{aligned}$$

問題③

$$\int \sin 3x \cos 2x \, dx$$

1. 次の不定積分を求めよ。

Find the following indefinite integral.

$$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$$

例題①

$$\begin{aligned} \int x e^{-x} dx &= \int x (-e^{-x})' dx \\ &= x (-e^{-x}) - \int (x)' (-e^{-x}) dx \\ &= -x e^{-x} - e^{-x} + C \end{aligned}$$

問題①

$$\int x e^x dx$$

例題②

$$\begin{aligned} \int x^2 e^{-x} dx &= \int x^2 (-e^{-x})' dx \\ &= -x^2 e^{-x} - \int (x^2)' (-e^{-x}) dx \\ &= -x^2 e^{-x} + 2 \int x e^{-x} dx \\ &= -x^2 e^{-x} - 2 x e^{-x} - 2 e^{-x} + C \end{aligned}$$

問題②

$$\int x^2 e^x dx$$

例題③

$$\begin{aligned} \int \log x dx &= \int \log x \times (x)' dx \\ &= x \log x - \int (\log x)' x dx \\ &= x \log x - \int dx \\ &= x \log x - x + C \end{aligned}$$

問題③

$$\int (\log x)^2 dx$$

2. 次の不定積分を求めよ。

Find the following indefinite integral.

$$\sin^2 \alpha = \frac{1 - \cos 2 \alpha}{2}, \quad \cos^2 \alpha = \frac{1 + \cos 2 \alpha}{2}$$

$$\sin \alpha \cos \beta = \frac{1}{2} \left\{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \right\}$$

$$\cos \alpha \cos \beta = \frac{1}{2} \left\{ \cos(\alpha + \beta) + \cos(\alpha - \beta) \right\}$$

$$\sin \alpha \sin \beta = -\frac{1}{2} \left\{ \cos(\alpha + \beta) - \cos(\alpha - \beta) \right\}$$

例題①

$$\begin{aligned} \int \cos^2 3 x dx \\ = \int \frac{1 + \cos 6 x}{2} dx = \frac{1}{2} x + \frac{1}{12} \sin 6 x + C \end{aligned}$$

問題①

$$\int \sin^2 4 x dx$$

例題②

$$\begin{aligned} \int \cos 3 x \cos x dx \\ = \int \frac{1}{2} \left\{ \cos(3 x + x) + \cos(3 x - x) \right\} dx \\ = \frac{1}{2} \int (\cos 4 x + \cos 2 x) dx \\ = \frac{1}{2} \left( \frac{1}{4} \sin 4 x + \frac{1}{2} \sin 2 x \right) + C \\ = \frac{1}{8} \sin 4 x + \frac{1}{4} \sin 2 x + C \end{aligned}$$

問題②

$$\int \cos 3 x \cos 2 x dx$$

問題③

$$\int \sin 2 x \sin x dx$$

1. 次の三角関数を微分せよ。  
Differentiate the following trigonometric functions.

例題

$$\frac{1}{\tan x} = \frac{\cos x}{\sin x}$$
$$\left(\frac{1}{\tan x}\right)' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{\sin^2 x}$$
$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

問題

$\tan x$

2. 次の不定積分を求めよ。  
Find the following indefinite integral.

例題

$$\int \frac{x \, dx}{\sin^2 x}$$
$$\int \frac{x \, dx}{\sin^2 x} = \int x \left(-\frac{1}{\tan x}\right)' \, dx$$
$$= -\frac{x}{\tan x} - \int x' \left(-\frac{1}{\tan x}\right) \, dx$$
$$= -\frac{x}{\tan x} + \int \frac{1}{\tan x} \, dx$$
$$= -\frac{x}{\tan x} + \int \frac{(\sin x)'}{\sin x} \, dx$$
$$= -\frac{x}{\tan x} + \log |\sin x| + C$$

問題

$$\int \frac{x \, dx}{\cos^2 x}$$

3. 次の不定積分を求めよ。  
Find the following indefinite integral.

例題①

$$\int x^2 \log x \, dx = \int \left(-\frac{x^3}{3}\right)' \log x \, dx$$
$$= -\frac{x^3}{3} \log x - \int \frac{x^3}{3} (\log x)' \, dx$$
$$= -\frac{x^3}{3} \log x - \frac{1}{3} \int x^2 \, dx$$
$$= -\frac{x^3}{3} \log x - \frac{x^3}{9} + C$$

問題①

$$\int x \log x \, dx$$

例題②

$$\int (x + 1) e^{-x} \, dx = \int (x + 1) (-e^{-x})' \, dx$$
$$= -(x + 1) e^{-x} - \int (x + 1)' (-e^{-x}) \, dx$$
$$= -(x + 1) e^{-x} + \int e^{-x} \, dx$$
$$= -(x + 1) e^{-x} - e^{-x} + C = -(x + 2) e^{-x} + C$$

問題②

$$\int x e^x \, dx$$

問題③

$$\int x^2 e^x \, dx$$

1. 次の不定積分を求めよ。
2. 次の不定積分を求めよ。

Find the following indefinite integral.

Find the following indefinite integral.

れい　だい

例題

①

$$\int \frac{\log x}{x} dx$$

※ちかんせきぶん  
置換積分

$$\log x = t \text{ とおくと } \frac{dt}{dx} = \frac{1}{x} \text{ , } \frac{1}{x} dx = dt$$

$$\int \frac{\log x}{x} dx = \int t dt$$

$$= \frac{1}{2} t^2 + C = -\frac{1}{2} (\log x)^2 + C$$

もんだい

問題

①

$$\int \frac{(\log x)^2}{x} dx$$

※ちかんせきぶん  
置換積分

れい　だい

例題

②

$$\int \log x dx = \int (x)' \log x dx$$

$$= x \log x - \int x (\log x)' dx$$

$$= x \log x - \int x \left(\frac{1}{x}\right) dx$$

$$= x \log x - \int dx$$

$$= x \log x - x + C$$

もんだい

問題

②

$$\int \log (x + 3) dx = \int (x + 3)' \log (x + 3) dx$$

もんだい

問題

③

$$\int (\log x)^2 dx = \int (x)' (\log x)^2 dx$$

もんだい

問題

④

$$\int \frac{\log x}{x^2} dx = \int \left(-\frac{1}{x}\right)' \log x dx$$

れい　だい

例題

①

$$\int x e^{-x^2} dx$$

※ちかんせきぶん  
置換積分

$$u = -x^2 \text{ とおくと } \frac{du}{dx} = -2x \text{ , } du = -2x dx$$

$$\int x e^{-x^2} dx = -\frac{1}{2} \int e^{-x^2} (-2x) dx$$

$$= -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + C = -\frac{1}{2} e^{-x^2} + C$$

もんだい

問題

①

$$\int x e^{x^2} dx$$

※ちかんせきぶん  
置換積分

れい　だい

例題

②

$$\int x e^{3x} dx = \int x \left(\frac{1}{3} e^{3x}\right)' dx$$

$$= \frac{1}{3} x e^{3x} - \int (x)' \frac{1}{3} e^{3x} dx$$

$$= \frac{1}{3} x e^{3x} - \frac{1}{3} \int e^{3x} dx$$

$$= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C$$

もんだい

問題

②

$$\int x e^{-x} dx = \int x (-e^{-x})' dx$$

もんだい

問題

③

$$\int x^2 e^{-x} dx = \int x^2 (-e^{-x})' dx$$

1. 次の不定積分を求めよ。

Find the following indefinite integral.

2. 次の関数を微分せよ。

Differentiate the following function.

れいだい

例題①

$$\int x \sin 2 x \, dx = \int x \left( -\frac{1}{2} \cos 2 x \right)' dx$$
$$= x \left( -\frac{1}{2} \cos 2 x \right) - \int (x)' \left( -\frac{1}{2} \cos 2 x \right) dx$$
$$= x \left( -\frac{1}{2} \cos 2 x \right) - \int \left( -\frac{1}{2} \sin 2 x \right) dx$$
$$= -\frac{1}{2} x \cos 2 x + \frac{1}{4} \sin 2 x + C$$

もんだい

問題①

$$\int x \sin 3 x \, dx$$

れいだい

例題②

$$\int x \cos 4 x \, dx = \int x \left( \frac{1}{4} \sin 4 x \right)' dx$$
$$= x \left( \frac{1}{4} \sin 4 x \right) - \int (x)' \left( \frac{1}{4} \sin 4 x \right) dx$$
$$= x \left( \frac{1}{4} \sin 4 x \right) - \int \frac{1}{4} \cos 4 x \, dx$$
$$= \frac{1}{4} x \sin 4 x + \frac{1}{16} \cos 4 x + C$$

もんだい

問題②

$$\int x \cos 2 x \, dx$$

れいだい

例題③

$$\int x^2 \sin 4 x \, dx = \int x^2 \left( -\frac{1}{4} \cos 4 x \right)' dx$$
$$= x^2 \left( -\frac{1}{4} \cos 4 x \right) - \int (x^2)' \left( -\frac{1}{4} \cos 4 x \right) dx$$
$$= x^2 \left( -\frac{1}{4} \cos 4 x \right) + \frac{1}{2} \int x \cos 4 x \, dx$$
$$= -\frac{1}{4} x^2 \cos 4 x + \frac{1}{8} x \sin 4 x + \frac{1}{32} \cos 4 x + C$$

れいだい

例題③

$$\int x^2 \cos 2 x \, dx$$

れいだい

例題

①

$$y = \log |\sin x|$$
$$y' = \frac{(\sin x)'}{(\sin x)} = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

もんだい

問題

①

$$y = -\log |\cos x|$$

れいだい

例題

②

$$y = \frac{1}{\tan x}$$
$$y' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{(\sin x)^2}$$
$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

もんだい

問題

②

$$y = \tan x$$

3. 次の不定積分を求めよ。

Find the following indefinite integral.

れいだい

例題

$$\int \frac{x}{\sin^2 x} \, dx = \int x \left( -\frac{1}{\tan x} \right)' dx$$
$$= -\frac{x}{\tan x} - \int (x)' \left( -\frac{1}{\tan x} \right)' dx$$
$$= -\frac{x}{\tan x} + \int \frac{1}{\tan x} \, dx$$
$$= -\frac{x}{\tan x} + \log |\sin x| + C$$

もんだい

問題

$$\int \frac{x}{\cos^2 x} \, dx = \int x (\tan x)' dx$$

1. 次の不定積分を求めよ。

Find the following indefinite integral.
2. 次の不定積分を求めよ。

Find the following indefinite integral.

れいだい

例題①

$$\int x \sin (x+1) dx$$
$$\int x \sin (x+1) dx = \int x \{ -\cos (x+1) \}' dx$$
$$= -x \cos (x+1) - \int (x)' \{ -\cos (x+1) \} dx$$
$$= -x \cos (x+1) + \int \cos (x+1) dx$$
$$= -x \cos (x+1) + \sin (x+1) + C$$

もんだい

問題①

$$\int x \cos (x+1) dx$$

れいだい

例題②

$$\int \cos \sqrt{x} dx$$
$$t = \sqrt{x} \text{ とおくと, } x = t^2, \frac{dx}{dt} = 2t, dx = 2t dt$$
$$\int \cos \sqrt{x} dx = \int 2t \cos t dt = \int 2t (\sin t)' dt$$
$$= 2t \sin t - \int (2t)' \sin t dt$$
$$= 2t \sin t - 2 \int \sin t dt = 2t \sin t + 2 \cos t + C$$
$$= 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$$

もんだい

問題②

$$\int \sin \sqrt{x} dx$$

れいだい

例題①

$$\int e^{\sqrt{x}} dx$$
$$t = \sqrt{x} \text{ とおくと, } x = t^2, \frac{dx}{dt} = 2t, dx = 2t dt$$
$$\int e^{\sqrt{x}} dx = \int 2te^t dt = \int 2t(e^t)' dt$$
$$= 2te^t - \int (2t)'e^t dt$$
$$= 2te^t - 2 \int e^t dt = 2te^t - 2e^t + C$$
$$= 2e^{\sqrt{x}}(\sqrt{x}-1) + C$$

もんだい

問題①

$$\int e^{-\sqrt{x}} dx$$

もんだい

問題②

$$\int e^{\sqrt[3]{x}} dx$$

1. 次の分数式を簡単な形で表せ。  
Express the following fractional expression in simple form.
2. 次の不定積分を求めよ。  
Find the following indefinite integral.

例題①

$$\frac{2x^2+3x+1}{x+2}$$
$$\begin{array}{r} 2x^2+3x+1 \phantom{00} \\ x+2 \overline{) 2x^2+3x+1} \\ \underline{2x^2+4x} \phantom{00} \\ -x+1 \phantom{00} \\ \underline{-x-2} \phantom{00} \\ 3 \phantom{00} \end{array}$$
$$= (x+2) \left( 2x-1 \right) + 3$$
$$\frac{2x^2+3x+1}{x+2} = \left( 2x-1 \right) + \frac{3}{x+2}$$

問題①

$$\frac{2x^2-3x-1}{x-2}$$

例題②

$$\frac{2x+1}{x^3+2x^2+x}$$
$$\frac{2x+1}{x^3+2x^2+x} = \frac{2x+1}{x(x+1)^2}$$
$$= \frac{a}{x} + \frac{b}{x+1} + \frac{c}{(x+1)^2}$$
$$\frac{a(x^2+2x+1)+b(x^2+x)+cx}{x(x+1)^2}$$
$$a+b=0, 2a+b+c=2, a=1 \text{ より}$$
$$a=1, b=-1, c=1 \text{ になり}$$
$$\frac{2x+1}{x^3+2x^2+x} = \frac{1}{x} - \frac{1}{x+1} + \frac{1}{(x+1)^2}$$

問題②

$$\frac{2x+1}{x^3+2x^2}$$

例題①

$$\int \frac{2x^2+3x+1}{x+2} dx$$
$$= \int \left( 2x-1 + \frac{3}{x+2} \right) dx$$
$$= x^2 - x + 3 \log |x+2| + C$$

問題①

$$\int \frac{2x^2-3x-1}{x-2} dx$$

例題②

$$\int \frac{2x+1}{x^3+2x^2+x} dx$$
$$= \int \left( \frac{1}{x} - \frac{1}{x+1} + \frac{1}{(x+1)^2} \right) dx$$
$$= \log |x| - \log |x+1| - \frac{1}{x+1} + C$$

問題②

$$\int \frac{2x+1}{x^3+2x^2} dx$$

例題③

$$\int \frac{1}{(x+1)(x+2)} dx$$
$$= \int \left( \frac{1}{x+1} - \frac{1}{x+2} \right) dx$$
$$= \log |x+1| - \log |x+2| + C$$

問題③

$$\int \frac{1}{(x+2)(x+3)} dx$$

問題④

$$\int \frac{2}{x(x+2)} dx$$

1. 次の分数式を簡単な形で表せ。  
Express the following fractional expression in simple form.
2. 次の不定積分を求めよ。  
Find the following indefinite integral.

例題①

$$\frac{2x^2+5x-1}{x+3}$$

$$\begin{array}{r} 2x^2+5x-1 \qquad \qquad x+3 \overline{) 2x^2+5x-1} \\ \underline{2x^2+6x} \phantom{-1} \\ -x-1 \\ \underline{-x-3} \\ 2 \end{array}$$
$$= (x+3) \left( 2x-1 \right) + 2$$
$$\frac{2x^2+5x-1}{x+3} = \left( 2x-1 \right) + \frac{2}{x+3}$$

問題①

$$\frac{2x^2+4x+2}{x+2}$$

例題②

$$\frac{1}{x^3+x^2}$$

$$\frac{1}{x^3+x^2} = \frac{1}{x^2(x+1)} = \frac{a}{x^2} + \frac{b}{x} + \frac{c}{x+1}$$
$$a = \frac{1}{x^2(x+1)} \times x^2 \Big|_{x=0} = \frac{1}{(0+1)} = 1$$
$$c = \frac{1}{x^2(x+1)} \times (x+1) \Big|_{x=-1} = \frac{1}{(-1)^2} = 1$$

$x=1$  のとき,

$$\frac{1}{1^3+1^2} = \frac{1}{1^2} + \frac{b}{1} + \frac{1}{1+1} \quad \text{より } b = -1$$
$$\frac{1}{x^3+x^2} = \frac{1}{x^2} - \frac{1}{x} + \frac{1}{x+1}$$

問題②

$$\frac{4}{x^3+2x^2}$$

例題①

$$\int \frac{2x^2+5x-1}{x+3} dx$$
$$= \int \left( 2x-1 + \frac{2}{x+3} \right) dx$$
$$= x^2 - x + 2 \log |x+3| + C$$

問題①

$$\int \frac{2x^2+4x+2}{x+2} dx$$

例題②

$$\int \frac{1}{x^3+x^2} dx$$
$$= \int \left( \frac{1}{x^2} - \frac{1}{x} + \frac{1}{x+1} \right) dx$$
$$= -\frac{1}{x} - \log |x| + \log |x+1| + C$$

問題②

$$\int \frac{4}{x^3+2x^2} dx$$

例題③

$$\int \frac{1}{(x+3)(x+4)} dx$$
$$= \int \left( \frac{1}{x+3} - \frac{1}{x+4} \right) dx$$
$$= \log |x+3| - \log |x+4| + C$$

問題③

$$\int \frac{1}{x(x+1)} dx$$

問題④

$$\int \frac{3}{x(x+3)} dx$$



1. 次の分数式を簡単な形で表せ。  
Express the following fractional expression in simple form.
2. 次の不定積分を求めよ。  
Find the following indefinite integral.

例題①

$$\frac{7x+8}{x^2+3x+2}$$
$$\frac{7x+8}{x^2+3x+2} = \frac{7x+8}{(x+1)(x+2)} = \frac{a}{x+1} + \frac{b}{x+2}$$
$$a = \frac{7x+8}{(x+1)(x+2)} \times (x+1) \Big|_{x=-1} = \frac{7(-1)+8}{(-1+2)} = 1$$
$$b = \frac{7x+8}{(x+1)(x+2)} \times (x+2) \Big|_{x=-2} = \frac{7(-2)+8}{(-2+1)} = 6$$
$$\frac{7x+8}{x^2+3x+2} = \frac{1}{x+1} + \frac{6}{x+2}$$

問題①

$$\frac{2x+1}{x^2-x}$$

例題②

$$\frac{x^3+2}{x^2+3x+2}$$
$$\begin{array}{r} x^3+2 \\ x^2+3x+2 \overline{) x^3 \phantom{+2} +2} \\ \underline{x^3+3x^2+2x} \phantom{+2} \\ -3x^2-2x+2 \phantom{+2} \\ \underline{-3x^2-9x-6} \\ 7x+8 \end{array}$$
$$\frac{x^3+2}{x^2+3x+2} = x-3 + \frac{7x+8}{x^2+3x+2}$$
$$= x-3 + \frac{1}{x+1} + \frac{6}{x+2}$$

例題②

$$\frac{x^3+x^2+1}{x^2-x}$$

例題①

$$\int \frac{x^3+2}{x^2+3x+2} dx$$
$$= \int \left( x-3 + \frac{1}{x+1} + \frac{6}{x+2} \right) dx$$
$$= \frac{x^2}{2} - 3x + 2 \log|x+1| + 6 \log|x+2| + C$$

問題①

$$\int \frac{x^3+x^2+1}{x^2-x} dx$$

問題②

$$\int \frac{2x^3+2x^2+6}{x^2+2x} dx$$

1. 次の分数式を簡単な形で表せ。  
Express the following fractional expression in simple form.
2. 次の不定積分を求めよ。  
Find the following indefinite integral.

例題①

$$\frac{2x^2-3}{x-2}$$
$$\begin{array}{r} 2x^2-3 \\ x-2 \overline{) 2x^2-4x+8} \\ \underline{4x-8} \end{array}$$
$$= (x-2)(2x+4) + 5$$
$$\frac{2x^2-3}{x-2} = (2x+4) + \frac{5}{x-2}$$

問題①

$$\frac{2x^2-4}{x-1}$$

例題②

$$\frac{2x+1}{x^3-2x^2+x}$$
$$\frac{2x+1}{x^3-2x^2+x} = \frac{2x+1}{x(x-1)^2}$$
$$= \frac{a}{x} + \frac{b}{x-1} + \frac{c}{(x-1)^2}$$
$$\frac{a(x^2-2x+1)+b(x^2-x)+cx}{x(x-1)^2}$$
$$a+b=0, -2a-b+c=2, a=1 \text{ より}$$
$$a=1, b=-1, c=3 \text{ になり}$$
$$\frac{2x+1}{x^3-2x^2+x} = \frac{1}{x} - \frac{1}{x-1} + \frac{3}{(x-1)^2}$$

問題②

$$\frac{3x+1}{x^3+2x^2+x}$$

例題①

$$\int \frac{2x^2-3}{x-2} dx$$
$$= \int \left( 2x+4 + \frac{5}{x-2} \right) dx$$
$$= x^2 + 4x + 5 \log |x-2| + C$$

問題①

$$\int \frac{2x^2-4}{x-1} dx$$

例題②

$$\int \frac{2x+1}{x^3-2x^2+x} dx$$
$$= \int \left( \frac{1}{x} - \frac{1}{x-1} + \frac{3}{(x-1)^2} \right) dx$$
$$= \log |x| - \log |x-1| - \frac{3}{x-1} + C$$

問題②

$$\int \frac{3x+1}{x^3+2x^2+x} dx$$

例題③

$$\int \frac{1}{(x+1)(x+3)} dx$$
$$= \frac{1}{2} \int \left( \frac{1}{x+1} - \frac{1}{x+3} \right) dx$$
$$= \frac{1}{2} \left( \log |x+1| - \log |x+3| \right) + C$$

問題③

$$\int \frac{1}{(x+1)(x+4)} dx$$

1. 次の分数式を簡単な形で表せ。  
Express the following fractional expression in simple form.
2. 次の不定積分を求めよ。  
Find the following indefinite integral.

例題①

$$\frac{2x^2+3x+2}{x+1}$$

$$\begin{array}{r} 2x^2+3x+2 \qquad \qquad x+1 \overline{) 2x^2+3x+2} \\ \underline{2x^2+2x} \phantom{+2} \\ x+2 \phantom{+2} \\ \underline{x+1} \phantom{+2} \\ 1 \phantom{+2} \end{array}$$

$$= (x+1)(2x+1) + 1$$

$$\frac{2x^2+3x+2}{x+1} = (2x+1) + \frac{1}{x+1}$$

問題①

$$\frac{2x^2+4x+3}{x+1}$$

例題②

$$\frac{1}{x(x+1)^2} = \frac{a}{x} + \frac{b}{(x+1)^2} + \frac{c}{x+1}$$

$$a = \frac{1}{x(x+1)^2} \times x \Big|_{x=0} = \frac{1}{0(0+1)^2} = 1$$

$$b = \frac{1}{x(x+1)^2} \times (x+1)^2 \Big|_{x=-1} = \frac{1}{-1} = -1$$

$$x=1 \text{ のとき,}$$

$$\frac{1}{1(1+1)^2} = \frac{1}{1} + \frac{-1}{(1+1)^2} + \frac{c}{1+1} \quad c = -1$$

$$\frac{1}{x(x+1)^2} = \frac{1}{x} - \frac{1}{(x+1)^2} - \frac{1}{x+1}$$

問題②

$$\frac{1}{x(x-1)^2} = \frac{a}{x} + \frac{b}{(x-1)^2} + \frac{c}{x-1}$$

例題①

$$\int \frac{2x^2+3x+2}{x+1} dx$$

$$= \int \left( 2x+2 + \frac{1}{x+1} \right) dx$$

$$= x^2 + 2x + \log|x+1| + C$$

問題①

$$\int \frac{2x^2+4x+3}{x+1} dx$$

例題②

$$\int \frac{1}{x(x+1)^2}$$

$$= \int \left( \frac{1}{x} - \frac{1}{(x+1)^2} - \frac{1}{x+1} \right)$$

$$= \log|x| + \frac{1}{x+1} - \log|x+1| + C$$

問題②

$$\int \frac{1}{x(x+1)^2} x$$

例題③

$$\int \frac{1}{x(x-3)} dx$$

$$= \frac{1}{3} \int \left( \frac{1}{x-3} - \frac{1}{x} \right) dx$$

$$= \frac{1}{3} \left( \log|x-3| - \log|x| \right) + C$$

問題③

$$\int \frac{1}{x(x-2)} dx$$

1. 次の三角関数を  $\cos 2x$  を用いて表せ。  
Express the following trigonometric functions using  $\cos 2x$ .

れいだい

例題

$\sin^2 x$

$$\cos 2x = \cos(x+x) = \cos^2 x - \sin^2 x$$
$$= (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$$

よって、 $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

もんだい

問題

$\cos^2 x$

2. 次の積和公式を作れ。  
Derive the following product to sum formula.

れいだい

例題①

$\sin \alpha \cos \beta$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad \cdots \textcircled{1}$$
$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \quad \cdots \textcircled{2}$$

①+②より

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta$$
$$\sin \alpha \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

もんだい

問題①

$\cos \alpha \sin \beta$

れいだい

例題②

$\sin \alpha \sin \beta$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad \cdots \textcircled{1}$$
$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \quad \cdots \textcircled{2}$$

①-②より

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2 \sin \alpha \sin \beta$$
$$\sin \alpha \sin \beta = -\frac{1}{2} \{ \cos(\alpha + \beta) - \cos(\alpha - \beta) \}$$

もんだい

問題②

$\cos \alpha \cos \beta$

3. 次の不定積分を求めよ。  
Find the following indefinite integral.

れいだい

例題①

$\int (\sin 2x + \cos 3x + 1) dx$

$$= -\frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x + x + C$$

もんだい

問題①

$\int (\sin 4x + \cos 6x + 1) dx$

れいだい

例題②

$\int \cos^2 x \sin x dx$

$$u = \cos x \text{ とおくと } du = -\sin x dx$$
$$\int \cos^2 x \sin x dx = -\int \cos^2 x (-\sin x) dx$$
$$= -\int u^2 du = -\frac{1}{3} u^3 + C = -\frac{1}{3} \cos^3 x + C$$

もんだい

問題②

$\int \cos^3 x \sin x dx$

れいだい

例題③

$\int \cos^2 2x dx$

$$= \frac{1}{2} \int (1 + \cos 4x) dx$$
$$= \frac{1}{2} x + \frac{1}{8} \sin 4x + C$$

もんだい

問題④

$\int \sin^2 3x dx$

れいだい

例題⑤

$\int \sin 4x \cos x dx$

$$= \frac{1}{2} \int (\sin 5x + \sin 3x) dx$$
$$= -\frac{1}{10} \cos 5x - \frac{1}{6} \cos 3x + C$$

もんだい

問題⑤

$\int \sin 3x \sin x dx$

1. 次の関数を微分せよ。Differentiate the following function.
4. 次の不定積分を求めよ。Find the following indefinite integral.

例題

$$y = \tan x$$
$$y' = \frac{(\sin x)' \cos x - \sin x (\cos x)'}{(\cos x)^2}$$
$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

問題

$$y = \frac{1}{\tan x}$$

2. 次の等式を証明せよ。Prove the following equation.

例題

$$\frac{1}{\tan^2 x + 1} = \cos^2 x$$
$$\frac{1}{\tan^2 x + 1} = \frac{1}{\left(\frac{\sin x}{\cos x}\right)^2 + 1}$$
$$= \frac{\cos^2 x}{\sin^2 x + \cos^2 x} = \cos^2 x \quad \text{Q.E.D.}$$

問題

$$\tan^2 x + 1 = \frac{1}{\cos^2 x}$$

3. 次の不定積分を求めよ。Find the following indefinite integral.

例題

$$\int \frac{1}{\cos^2 x} dx$$
$$= \tan x + C$$

問題①

$$\int \frac{1}{\sin^2 x} dx$$

問題②

$$\int \tan^2 x dx$$

例題①

$$\int \cos^3 x dx$$
$$= \int \cos^2 x \cos x dx = \int (1 - \sin^2 x) \cos x dx$$
$$u = \sin x \text{ とおくと } du = \cos x dx$$
$$\int (1 - \sin^2 x) \cos x dx = \int (1 - u^2) du$$
$$= u - \frac{1}{3} u^3 + C = \sin x - \frac{1}{3} \sin^3 x + C$$

問題①

$$\int \sin^3 x dx$$

例題②

$$\int \frac{1}{\sin x} dx$$
$$= \int \frac{\sin x}{\sin^2 x} dx = \int \frac{\sin x}{1 - \cos^2 x} dx$$
$$u = \cos x \text{ とおくと } -\sin x dx = du$$
$$\int \frac{\sin x dx}{1 - \cos^2 x} = \int \frac{-\sin x dx}{\cos^2 x - 1} = \int \frac{du}{u^2 - 1}$$
$$= \frac{1}{2} \int \left( \frac{1}{u - 1} - \frac{1}{u + 1} \right) du$$
$$= \frac{1}{2} \left( \log |\cos x - 1| - \log |\cos x + 1| \right) + C$$
$$= \frac{1}{2} \log \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$$

問題②

$$\int \frac{1}{\cos x} dx$$

1. 次の関数を微分せよ。 Differentiate the following function.

例題

$y = \log \left| \cos x \right|$

$$y' = \frac{(\cos x)'}{(\cos x)} = \frac{-\cos x}{\sin x} = -\tan x$$

問題

$y = \log \left| \sin x \right|$

2. 次の不定積分を求めよ。 Find the following indefinite integral.

例題

$\int \tan x \, dx$

$$= -\log \left| \cos x \right| + C$$

問題

$\int \frac{1}{\tan x} \, dx$

3. 次の不定積分を求めよ。 Find the following indefinite integral.

例題①

$$\begin{aligned} \int x \sin 3x \, dx &= \int x \left( -\frac{1}{3} \cos 3x \right)' dx \\ &= -\frac{1}{3} x \cos 3x - \int (x)' \left( -\frac{1}{3} \cos 3x \right) dx \\ &= -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + C \end{aligned}$$

問題①

$$\int x \cos 3x \, dx$$

例題②

$$\begin{aligned} \int x^2 \cos 3x \, dx &= \int x^2 \left( \frac{1}{3} \sin 3x \right)' dx \\ &= \frac{1}{3} x^2 \sin 3x - \int (x^2)' \left( \frac{1}{3} \sin 3x \right) dx \\ &= \frac{1}{3} x^2 \sin 3x - \frac{2}{3} \int x \sin 3x \, dx \\ &= \frac{1}{3} x^2 \sin 3x + \frac{2}{9} x \cos 3x - \frac{2}{27} \sin 3x + C \end{aligned}$$

問題②

$$\int x^2 \sin 3x \, dx$$

4. 次の三角関数を  $\cos 2x$  を用いて表せ。 Express the following trigonometric functions using  $\cos 2x$ .

例題

$\cos^2 x$

$$\begin{aligned} \cos 2x &= \cos(x+x) = \cos^2 x - \sin^2 x \\ &= \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1 \end{aligned}$$

よって、 $\cos^2 x = \frac{1}{2} (1 + \cos 2x)$

問題

$\sin^2 x$

5. 次の不定積分を求めよ。 Find the following indefinite integral.

例題

$$\begin{aligned} \int \cos^4 x \, dx &= \int (\cos^2 x)^2 \, dx = \int \left\{ \frac{1}{2} (1 + \cos 2x) \right\}^2 \, dx \\ &= \frac{1}{4} \int (1 + 2\cos 2x + \cos^2 2x) \, dx \\ &= \frac{1}{4} \int \left\{ 1 + 2\cos 2x + \frac{1}{2} (1 + \cos 4x) \right\} \, dx \\ &= \frac{1}{8} \int (3 + 4\cos 2x + \cos 4x) \, dx \\ &= \frac{3}{8} x + \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C \end{aligned}$$

問題

$$\int \sin^4 x \, dx$$