数学Ⅲ　対数・指数関数の導関数 ①　課題

（　　）年（　　）組（　　）番（　　）

1．$\lim_{t \to 0}(1+t)^{\frac{1}{t}} = e$ より，次の極限を求めよ。

Find the limit value using $\lim_{t \to 0}(1+t)^{\frac{1}{t}} = e$.

例題	問題
$\lim_{n \to \infty}\left(1+\frac{2}{n}\right)^n$ $\frac{2}{n} = t$ とおく。 $n \to \infty$ のとき $t \to 0$ $\lim_{n \to \infty}\left(1+\frac{2}{n}\right)^n$ $= \lim_{t \to \infty}\left(1+t\right)^{\frac{2}{t}}$ $= \lim_{t \to \infty}\left\{\left(1+t\right)^{\frac{1}{t}}\right\}^2$ $= e^2$	$\lim_{n \to \infty}\left(1+\frac{1}{2n}\right)^n$

2．次の極限を求めよ。

Find the limit value.

例題	問題
$\lim_{x \to 0}\frac{\log(x+1)}{x}$ $f(x) = \log(x+1)$ とおく。 $f(0) = \log 1 = 0$ $\lim_{x \to 0}\frac{\log(x+1)}{x}$ $= \lim_{x \to 0}\frac{\log(x+1) - 0}{x - 0}$ $= \lim_{x \to 0}\frac{f(x) - f(0)}{x - 0}$ $= f'(0) = \frac{1}{0+1} = 1$	$\lim_{x \to 0}\frac{e^x - 1}{x}$

3．次の関数を微分せよ。　※ $\log_e x$ を $\log x$ と書く。

Differentiate the following function.

例題	問題
① $y = \log_2 x$ $y' = \frac{1}{x \log 2}$	① $y = \log_3 x$
② $y = \log 5x$ $y' = \frac{1}{5x} \times (5x)'$ $= \frac{1}{x}$	② $y = \log 4x$
③ $y = \log(x^2+3)$ $y' = \frac{1}{x^2+3} \times (x^2+3)'$ $= \frac{2x}{x^2+3}$	③ $y = \log(x+2)$
④ $y = 4^x$ $y' = 4^x \log 4$	④ $y = 3^x$
⑤ $y = 2^{3x}$ $y' = 2^{3x} \log 2 \times (3x)'$ $= 3 \times 2^{3x} \log 2$	⑤ $y = 3^{2x}$
⑥ $y = e^{4x}$ $y' = e^{4x} \times (4x)'$ $= 4e^{4x}$	⑥ $y = e^{3x}$
⑦ $y = x^2 \log x$ $y' = (x^2)' \log x + x^2 (\log x)'$ $= 2x \log x + x^2 \times \frac{1}{x}$ $= 2x \log x + x$	⑦ $y = x \log x$
⑧ $y = x^3 e^x$ $y' = (x^3)' e^x + x^3 (e^x)'$ $= 3x^2 e^x + x^3 e^x$ $= (x^3 + 3x^2) e^x$	⑧ $y = x^2 e^x$

数学Ⅲ　対数・指数関数の導関数 ②　課題　　(　　)年(　　)組(　　)番(　　)

1．$\lim_{h \to 0}(1+h)^{\frac{1}{h}} = e$ より，次の極限を求めよ。

Find the limit value using $\lim_{h \to 0}(1+h)^{\frac{1}{h}} = e$.

例題	問題
$\lim_{h \to 0}\left(1+h\right)^{-\frac{2}{h}}$ $= \lim_{h \to 0} \dfrac{1}{\left\{\left(1+h\right)^{\frac{1}{h}}\right\}^{2}}$ $= \dfrac{1}{e^{2}}$	$\lim_{h \to 0}\left(1+h\right)^{-\frac{1}{2h}}$

2．次の文章の［　　］を埋めて，$\log_a x$ を微分せよ。

a は 1 でない正の定数とする。

Fill in the blanks in the following sentences and differentiate $\log_a x$.

$$\lim_{h \to 0} \frac{\log_a(x+h) - \log_a x}{h}$$

$$= \lim_{h \to 0} \boxed{\quad} \log_a \boxed{\qquad\qquad}$$

$$= \lim_{h \to 0} \boxed{\quad} \log_a \left(1 + \boxed{\quad}\right)$$

$\dfrac{h}{x} = t$ とおくと　$h \to 0$ のとき，$t \to$［　　］

$$\lim_{h \to 0} \boxed{\quad} \log_a \left(1 + \boxed{\quad}\right)$$

$$= \lim_{t \to 0} \boxed{\quad} \log_a \left(1 + \boxed{\quad}\right)$$

$$= \lim_{t \to 0} \boxed{\quad} \log_a \left(1 + \boxed{\quad}\right)^{\frac{1}{h}} = \boxed{\quad} \log_a \boxed{\quad}$$

底の変換公式より　$\log_a e = \dfrac{\log_e e}{\log_e} = \dfrac{1}{\log}$

$$\left(\log_a x\right)' = \boxed{\qquad\qquad}$$

3．次の文章の［　　］を埋めて，$y = a^x$ を微分せよ。

Fill in the blanks in the following sentences and differentiate a^x.

$y = a^x$ の両辺を a を底とする対数をとると

$$\log_a y = \boxed{\log_a \quad} = \boxed{\log_a a} = x$$

$x = \log_a y$ を微分すると　$\dfrac{dx}{dy} = \boxed{\qquad\qquad}$

$$\frac{dy}{dx} = \frac{1}{\dfrac{dx}{dy}} = \frac{1}{\boxed{\qquad\qquad}} = \boxed{\quad}$$

4．次の関数を微分せよ。　Differentiate the following function.

例題	問題
① $y = \log_{10} x$ $y' = \dfrac{1}{x \log 10}$	① $y = \log_4 x$
② $y = \log \lvert 3x + 1 \rvert$ $y' = \dfrac{1}{3x+1} \times (3x+1)'$ $= \dfrac{3}{3x+1}$	② $y = \log \lvert 2x + 1 \rvert$
③ $y = \log \lvert \sin x \rvert$ $y' = \dfrac{1}{\sin x} \times (\sin x)'$ $= \dfrac{\cos x}{\sin x}$	③ $y = \log \lvert \cos x \rvert$
④ $y = 2^x$ $y' = 2^x \log 2$	④ $y = 8^x$
⑤ $y = e^{4x+1}$ $y' = e^{4x+1} \times (4x+1)'$ $= 4e^{4x+1}$	⑤ $y = e^{3x+1}$
⑥ $y = \dfrac{e^x}{x^2}$ ※商の公式 $y' = \dfrac{(e^x)' x^2 - e^x (x^2)'}{x^4}$ $= \dfrac{x^2 e^x - 2x e^x}{x^4}$ $= \dfrac{x e^x - 2e^x}{x^3}$ ※積の公式 $y' = (e^x)' x^{-2} + e^x (x^{-2})'$ $= x^{-2} e^x - 2x^{-3} e^x$ $= \dfrac{x e^x - 2e^x}{x^3}$	⑥ $y = \dfrac{e^x}{x}$

数学Ⅲ　対数・指数関数の導関数 ③ 課題　　(　　)年(　　)組(　　)番(　　)

1．$\lim_{h \to 0}(1+h)^{\frac{1}{h}} = e$ より，次の極限を求めよ。

Find the limit value using $\lim_{h \to 0}(1+h)^{\frac{1}{h}} = e$.

例題　$\lim_{x \to \infty}\left(1 - \frac{3}{x}\right)^{x}$

$-\frac{2}{x} = h$ とすると　$x = -\frac{3}{h}$

$x \to \infty$ のとき　$h \to 0$ より

$$\lim_{x \to \infty}\left(1 - \frac{3}{x}\right)^{x} = \lim_{h \to 0}\left(1+h\right)^{-\frac{3}{h}}$$

$$= \lim_{h \to 0}\frac{1}{\left\{\left(1+h\right)^{\frac{1}{h}}\right\}^{3}} = \frac{1}{e^{3}}$$

問題　$\lim_{x \to \infty}\left(1 + \frac{1}{2x}\right)^{x}$

2．次の文章の［　　］を埋めて，$\log_a x$ を微分せよ。

a は 1 でない正の定数とする。

Fill in the blanks in the following sentences and differentiate $\log_a x$.

$$\lim_{h \to 0}\frac{\log_a(x+h) - \log_a x}{h}$$

$$= \lim_{h \to 0}\boxed{\quad}\log_a\boxed{\quad\quad}$$

$$= \lim_{h \to 0}\boxed{\quad}\log_a\left(1 + \boxed{\quad\quad}\right)$$

$\frac{h}{x} = t$ とおくと　$h \to 0$ のとき，$t \to$［　　］

$$\lim_{h \to 0}\boxed{\quad}\log_a\left(1 + \boxed{\quad\quad}\right)$$

$$= \lim_{t \to 0}\boxed{\quad}\log_a\left(1 + \boxed{\quad\quad}\right)$$

$$= \lim_{t \to 0}\boxed{\quad}\log_a\left(1 + \boxed{\quad\quad}\right)^{\frac{1}{h}} = \boxed{\quad}\log_a\boxed{\quad}$$

底の変換公式より　$\log_a e = \frac{\log_e}{\log_e} = \frac{1}{\log}$

$$\left(\log_a x\right)' = \boxed{\quad\quad\quad}$$

3．次の文章の［　　］を埋めて，$y = a^x$ を微分せよ。

Fill in the blanks in the following sentences and differentiate a^x.

$y = a^x$ の両辺を a を底とする対数をとると

$\log_a y =$［$\log_a$　　］$=$［　　$\log_a a$］$= x$

$x = \log_a y$ を微分すると　$\frac{dx}{dy} =$［　　］

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\boxed{\quad\quad}} = \boxed{\quad\quad}$$

4．次の関数を微分せよ。　Differentiate the following function.

例題	問題
① $y = 3^x$ $y' = 3^x \log 3$	① $y = 5^x$
② $y = \log_3 x$ $y' = \frac{1}{x \log 3}$	② $y = \log_5 x$
③ $y = \log\lvert 2 - x\rvert$ $y' = \frac{1}{2-x} \times (2-x)'$ $= \frac{-1}{2-x} = \frac{1}{x-2}$	③ $y = \log\lvert 1 - x\rvert$
④ $y = e^{2x-1}$ $y' = e^{2x-1} \times (2x-1)'$ $= 2e^{2x-1}$	④ $y = e^{3x+1}$
⑤ $y = \frac{x^2}{e^x}$ ※商の公式 $y' = \frac{(x^2)'e^x - x^2(e^x)'}{e^{2x}}$ $= \frac{2xe^x - x^2e^x}{e^{2x}}$ $= \frac{2x - x^2}{e^x}$	⑤ $y = \frac{x}{e^x}$

数学Ⅲ　対数関数の導関数　課題

（　　）年（　　）組（　　）番（　　）

1．次の計算をせよ。　Calculate the following expression.

例題 $\left(1+\dfrac{x}{\sqrt{x^2-1}}\right)\left(x-\sqrt{x^2-1}\right)$

$=\left(\dfrac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}}\right)\left(x-\sqrt{x^2-1}\right)=\dfrac{1}{\sqrt{x^2-1}}$

問題 $\left(1+\dfrac{x}{\sqrt{x^2+1}}\right)\left(x-\sqrt{x^2+1}\right)$

2．次の式を微分せよ。　Differentiate the following expression.

例題①　$y=\log\left(x^2+1\right)$

$y'=\dfrac{1}{x^2+1}\times\left(x^2+1\right)'=\dfrac{2x}{x^2+1}$

問題①　$y=\log\left(x^2-1\right)$

例題②　$y=\log\left(\sin x\right)$

$y'=\dfrac{1}{\sin x}\times\left(\sin x\right)'=\dfrac{\cos x}{\sin x}=\dfrac{1}{\tan x}$

問題②　$y=\log\left(\cos x\right)$

例題③　$y=\dfrac{1}{3}\left(\log x\right)^3$

$y'=\dfrac{1}{3}\times 3\times\left(\log x\right)^2\dfrac{1}{x}=\dfrac{\left(\log x\right)^2}{x}$

問題③　$y=\dfrac{1}{2}\left(\log x\right)^2$

例題④　$y=x\log x+x$

$y'=\left(x\right)'\log x+x\left(\log x\right)'+\left(x\right)'$

$=\log x+x\times\dfrac{1}{x}+1=\log x+2$

問題④　$y=x\log x-x$

3．次の式を微分せよ。　Differentiate the following expression.

例題①　$y=x+\sqrt{x^2-1}$

$y'=1+\dfrac{1}{2}\times 2x\times\dfrac{1}{\sqrt{x^2-1}}$

$=1+\dfrac{x}{\sqrt{x^2-1}}$

問題①　$y=x+\sqrt{x^2+1}$

例題②　$y=\log\left(x+\sqrt{x^2-1}\right)$

$y'=\dfrac{1}{x+\sqrt{x^2-1}}\times\left(x+\sqrt{x^2-1}\right)'$

$=\left(x-\sqrt{x^2-1}\right)\left(1+\dfrac{x}{\sqrt{x^2-1}}\right)=\dfrac{1}{\sqrt{x^2-1}}$

問題②　$y=\log\left(x+\sqrt{x^2+1}\right)$

例題③　$y=\dfrac{\log x}{x^2}$

$y'=\dfrac{\left(\log x\right)'\times x^2-\log x\times\left(x^2\right)'}{\left(x^2\right)^2}$

$=\dfrac{x-2x\log x}{x^4}=\dfrac{1-2\log x}{x^3}$

問題③　$y=\dfrac{\log x}{x^3}$

数学Ⅲ　指数関数の導関数　課題

（　　）年（　　）組（　　）番（　　）

1. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ より，次の極限を求めよ。
Find the next limit.

例題

$$\lim_{x\to 0}\frac{\log(1+2x)}{x}$$

$$= \lim_{x\to 0}\log(1+2x)^{\frac{1}{x}}$$

$n = \frac{1}{2x}$ とおくと

$x \to +0$ のとき $n \to \infty$

$$\lim_{x\to 0}\log(1+2x)^{\frac{1}{x}}$$

$$= \lim_{n\to\infty}\log\left(1+\frac{1}{n}\right)^{\frac{n}{2}}$$

$$= \lim_{n\to\infty}\log\left\{\left(1+\frac{1}{n}\right)^n\right\}^{\frac{1}{2}}$$

$$= \log e^{\frac{1}{2}} = \frac{1}{2}$$

問題

$$\lim_{x\to 0}\frac{\log(1+x)}{2x}$$

2. 定義に従って，次の関数を微分せよ。
Differentiate the following function according to the definition.

例題 $f(x) = 2^x$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{2^{x+h}-2^x}{h}$$

$$= \lim_{h\to 0}\frac{2^x(2^h-1)}{h} = 2^x \times \lim_{h\to 0}\frac{2^h-1}{h}$$

$y = 2^h - 1$ とおくと $h = \log_2(1+y)$

$$\lim_{h\to 0}\frac{2^h-1}{h} = \lim_{y\to 0}\frac{y}{\log_2(1+y)}$$

$$= \lim_{y\to 0}\frac{1}{\frac{\log_2(1+y)}{y}} = \frac{1}{\log_2 e} = \log 2$$

ゆえに $f'(x) = 2^x \log 2$

問題 $f(x) = e^x$

3. 次の関数を微分せよ。
Differentiate the following function.

例題① $y = 3^x$

$$y' = 3^x \log 3$$

問題① $y = 5^x$

例題② $y = 3^{x+1} = 3 \times 3^x$

$$y' = 3^{x+1}\log 3 \times (x+1)'$$

$$= 3^{x+1}\log 3$$

問題② $y = 2^{x+1}$

例題③ $y = e^{2x+1}$

$$y' = e^{2x+1} \times (2x+1)'$$

$$= e^{2x+1} \times 2 = 2e^{2x+1}$$

問題③ $y = e^{-x+1}$

例題④ $y = \frac{e^x}{x}$

$$y' = \left(e^x\right)' \times \frac{1}{x} + e^x \times \left(\frac{1}{x}\right)'$$

$$= e^x \times \frac{1}{x} + e^x \times \left(-\frac{1}{x^2}\right)$$

$$= \frac{e^x(x-1)}{x^2}$$

問題④ $y = \frac{e^x}{x^2}$

数学Ⅲ　対数微分法　課題

（　　）年（　　）組（　　）番（　　）

1. $\{\log|f(x)|\}' = \dfrac{f'(x)}{f(x)}$ を用いて，関数を微分せよ。
Differentiate the function

例題　$y = \dfrac{x^2(x+1)}{x+2}$

$\log|y| = 2\log|x| + \log|x+1| - \log|x+2|$

両辺を x で微分して

$$\frac{y'}{y} = \frac{2}{x} + \frac{1}{x+1} - \frac{1}{x+2}$$

$$= \frac{2(x+1)(x+2) + x(x+2) - x(x+1)}{x(x+1)(x+2)}$$

$$= \frac{2x^2+7x+4}{x(x+1)(x+2)}$$

$$y' = \frac{2x^2+7x+4}{x(x+1)(x+2)} \times \frac{x^2(x+1)}{x+2}$$

$$= \frac{(2x^2+7x+4)x}{(x+2)^2}$$

問題　$y = \dfrac{x^2(x+3)}{x+1}$

2. 対数微分法により，次の関数を微分せよ。
Differentiate the following function using logarithmic differentiation.

例題①　$y = x^{\sin x}$　　$(x > 0)$

$$\log y = \log x^{\sin x} = \sin x \times \log x$$

$$\frac{y'}{y} = (\sin x)' \times \log x + \sin x \times (\log x)'$$

$$= \cos x \times \log x + \sin x \times \frac{1}{x}$$

$$y' = \left(\cos x \times \log x + \sin x \times \frac{1}{x}\right) \times x^{\sin x}$$

問題①　$y = x^{\cos x}$　　$(x > 0)$

例題②　$y = \dfrac{x}{\sqrt{x+1}}$

$$\log|y| = \log\left|\frac{x}{\sqrt{x+1}}\right|$$

$$= \log|x| - \frac{1}{2}\log|x+1|$$

$$\frac{y'}{y} = \frac{1}{x} - \frac{1}{2(x+1)} = \frac{x+2}{2x(x+1)}$$

$$y' = \frac{x+2}{2x(x+1)} \times \frac{x}{\sqrt{x+1}}$$

$$y' = \frac{x+2}{2(x+1)\sqrt{x+1}}$$

問題②　$y = \dfrac{2x}{\sqrt{x-1}}$