

1. $\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$ を利用して、次の極限を求めよ。
Find the limit value using $\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$.

例題

$$\lim_{h \rightarrow 0} \frac{1 - \cos h}{h^2}$$
$$= \lim_{h \rightarrow 0} \frac{(1 - \cos h)(1 + \cos h)}{h^2 (1 + \cos h)}$$
$$= \lim_{h \rightarrow 0} \frac{(1 - \cos^2 h)}{h^2 (1 + \cos h)} = \lim_{h \rightarrow 0} \frac{\sin^2 h}{h^2 (1 + \cos h)}$$
$$= \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right)^2 \times \frac{1}{1 + \cos h} = \frac{1}{2}$$

問題

$$\lim_{h \rightarrow 0} \frac{\cos h - 1}{h}$$

2. 次の文章の を埋めて、 $\sin x$ を微分せよ。
Fill in the blanks in the following sentences and differentiate $\sin x$.

$$\sin(x + h) - \sin x$$
$$= \sin \cos \cos \sin - \sin x$$
$$= (\quad) \sin x + \quad \cos x$$
$$\lim_{h \rightarrow 0} \frac{\sin(x + h) - \sin x}{h}$$
$$= \lim_{h \rightarrow 0} \left(\frac{ \quad }{h} \sin x + \frac{ \quad }{h} \cos x \right)$$
$$\lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = \quad, \quad \lim_{h \rightarrow 0} \frac{\sin h}{h} = \quad \text{より}$$
$$\lim_{h \rightarrow 0} \frac{\sin(x + h) - \sin x}{h}$$
$$= \quad \times \sin x + \quad \times \cos x = \quad$$

よって、 $(\sin x)' = \quad$

3. 次の文章の を埋めて、 $\cos x$ を微分せよ。
Fill in the blanks in the following sentences and differentiate $\cos x$.

$$\sin \left(x + \frac{\pi}{2} \right)$$
$$= \sin x \cos \frac{\pi}{2} + \cos \sin$$
$$= \sin x \times \quad + \cos x \times \quad = \quad$$
$$\cos \left(x + \frac{\pi}{2} \right)$$
$$= \cos x \cos \frac{\pi}{2} - \sin \sin$$
$$= \cos x \times \quad - \sin x \times \quad = \quad$$

合成関数の微分法より

$$(\cos x)' = \left\{ \quad \left(x + \frac{\pi}{2} \right) \right\}'$$
$$= \quad \left(x + \frac{\pi}{2} \right) \left(x + \frac{\pi}{2} \right)'$$
$$= \quad$$

4. 次の三角関数を微分せよ。
Differentiate the following trigonometric function.

例題	問題
<div>① $y = \sin 2x$ $y' = \cos 2x \times (2x)'$ $= 2 \cos 2x$</div>	<div>① $y = \sin 4x$</div>
<div>② $y = \cos 3x$ $y' = -\sin 3x \times (3x)'$ $= -3 \sin 3x$</div>	<div>② $y = \cos 5x$</div>
<div>③ $y = \sin^2 x = (\sin x)^2$ $y' = 2 \sin x \times (\sin x)'$ $= 2 \sin x \cos x$</div>	<div>③ $y = \cos^2 x = (\cos x)^2$</div>
<div>④ $y = \frac{1}{\cos x}$ $y' = -\frac{(\cos x)'}{\cos^2 x}$ $= \frac{\sin x}{\cos^2 x}$</div>	<div>④ $y = \frac{1}{\sin x}$</div>

1. 次の文章の を埋めて、 $\cos x$ を微分せよ。

Fill in the blanks in the following sentences and differentiate $\cos x$.

$$\cos(x + h) - \cos x$$
$$= \cos \quad \cos \quad \sin \quad \sin \quad - \cos x$$
$$= \cos x (\quad) + \sin x (\quad)$$

$$\lim_{h \rightarrow 0} \frac{\cos(x + h) - \cos x}{h}$$
$$= \lim_{h \rightarrow 0} \left(\cos x \frac{ \quad }{h} + \sin x \frac{ \quad }{h} \right)$$

$$\lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = 0, \quad \lim_{h \rightarrow 0} \frac{\sin h}{h} = \quad \text{より}$$

$$\lim_{h \rightarrow 0} \frac{\cos(x + h) - \cos x}{h}$$
$$= \cos x \times \quad + \sin x \times \quad = \quad$$

$$\text{よって, } (\cos x)' = \quad$$

2. 次の文章の を埋めて、 $\sin x$, $\tan x$ を微分せよ。

Fill in the blanks in the following sentences and differentiate $\sin x$, $\tan x$.

$$\sin\left(x + \frac{\pi}{2}\right)$$
$$= \sin x \cos \frac{\pi}{2} + \cos \quad \sin \quad$$
$$= \sin x \times \quad + \cos x \times \quad = \quad$$

$$\cos\left(x + \frac{\pi}{2}\right)$$
$$= \cos x \cos \frac{\pi}{2} - \sin \quad \sin \quad$$
$$= \cos x \times \quad - \sin x \times \quad = \quad$$

合成関数の微分法より

$$(\sin x)' = \left\{ \quad \left(x + \frac{\pi}{2}\right) \right\}'$$
$$= \quad \left(x + \frac{\pi}{2}\right) \left(x + \frac{\pi}{2}\right)'$$
$$= \quad$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x} \right)' = \frac{(\sin x)' \cos x - (\sin x)' \cos x}{\cos^2 x}$$
$$= \frac{ \quad }{\cos^2 x}$$
$$= \quad$$

3. 次の三角関数を微分せよ。

Differentiate the following trigonometric function.

例題① $y = \sin(2x + 1)$

$$y' = \cos(2x + 1) \times (2x + 1)' = 2 \cos(2x + 1)$$

問題① $y = \sin(3x + 2)$

例題② $y = \cos(1 - 3x)$

$$y' = -\sin(1 - 3x) \times (1 - 3x)' = 3 \sin(1 - 3x)$$

問題② $y = \cos(2 - x)$

例題③ $y = \frac{1}{\sin x}$

$$y' = -\frac{(\sin x)'}{\sin^2 x} = -\frac{\cos x}{\sin^2 x}$$

問題③ $y = \frac{1}{\cos x}$

例題④ $y = x \sin x$

$$y' = x' \sin x + x (\sin x)' = \sin x + x \cos x$$

問題④ $y = x \cos x$

例題⑤ $y = \cos^3 x$

$$y' = 3 \cos^2 x (\cos x)' = -3 \cos^2 x \sin x$$

問題⑤ $y = \sin^3 x$

例題⑥ $y = \frac{\sin x}{x}$

$$y' = \frac{(\sin x)' x - \sin x \times x'}{x^2}$$
$$= \frac{x \cos x - \sin x}{x^2}$$

問題⑥ $y = \frac{\cos x}{x}$

1. 文章の を埋めて、和積公式を完成せよ。
Fill in the blanks in the sentences to complete the sum to product formulas.

$$\sin(\alpha + \beta) = \sin \quad \cos \quad + \cos \quad \sin \quad$$
$$\sin(\alpha - \beta) = \sin \quad \cos \quad - \cos \quad \sin \quad$$

両 辺の差をとり

$$\sin(\alpha + \beta) - \sin(\alpha - \beta) = \cos \quad \sin \quad$$
$$\alpha = \frac{A + B}{2} \quad , \quad \beta = \frac{A - B}{2} \quad \text{とおくと}$$
$$\alpha + \beta = \quad , \quad \alpha - \beta = \quad \text{より}$$

$$\sin \quad - \sin \quad = \cos \quad \sin \quad$$

$$\cos(\alpha + \beta) = \cos \quad \cos \quad - \sin \quad \sin \quad$$
$$\cos(\alpha - \beta) = \cos \quad \cos \quad + \sin \quad \sin \quad$$

両 辺の差をとり

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = \sin \quad \sin \quad$$
$$\alpha = \frac{A + B}{2} \quad , \quad \beta = \frac{A - B}{2} \quad \text{とおくと}$$
$$\alpha + \beta = \quad , \quad \alpha - \beta = \quad \text{より}$$

$$\cos \quad - \cos \quad = \sin \quad \sin \quad$$

2. 和積公式を用いて、次の式を求めよ。
Find the following equation usinng the sum to product formulas.

例題 $(\cos x)'$

$$= \lim_{h \rightarrow 0} \frac{\cos(x + h) - \cos x}{h}$$
$$= \lim_{h \rightarrow 0} \frac{-2 \sin(x + \frac{h}{2}) \sin \frac{h}{2}}{h}$$
$$= \lim_{h \rightarrow 0} -\sin(x + \frac{h}{2}) \times \frac{\sin \frac{h}{2}}{\frac{h}{2}} = -\sin x$$

問題 $(\sin x)'$

3. 次の文章の を埋めて、 $\sin x$ を微分せよ。
Fill in the blanks in the following sentences and differentiate $\sin x$.

$$\sin\left(x - \frac{\pi}{2}\right)$$
$$= \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin \quad$$
$$= \sin x \times \quad + \cos x \times \quad = \quad$$
$$\cos\left(x - \frac{\pi}{2}\right)$$
$$= \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin \quad$$
$$= \cos x \times \quad - \sin x \times \quad = \quad$$

合成関数の微分法より

$$(\sin x)' = \left\{ \quad \left(x - \frac{\pi}{2}\right) \right\}'$$
$$= \quad \left(x - \frac{\pi}{2}\right) \left(x - \frac{\pi}{2}\right)'$$
$$= \quad$$

4. 次の三角関数を微分せよ。
Differentiate the following trigonometric function.

例題	問題
<p>① $y = x \cos x$</p> $y' = (x)' \cos x + x (\cos x)'$ $= \cos x - x \sin x$	<p>① $y = x \sin x$</p>
<p>② $y = \sin 3 x$</p> $y' = \cos 3 x \times (3 x)'$ $= 3 \cos 3 x$	<p>② $y = \sin 4 x$</p>
<p>③ $y = \cos^2 x = (\cos x)^2$</p> $y' = 2 \cos x \times (\cos x)'$ $= -2 \sin x \cos x$	<p>③ $y = \sin^2 x = (\sin x)^2$</p>
<p>④ $y = \frac{1}{\cos 2 x}$</p> $y' = -\frac{(\cos 2 x)'}{\cos^2 2 x}$ $= \frac{2 \sin 2 x}{\cos^2 2 x}$	<p>④ $y = \frac{1}{\sin 2 x}$</p>