

1. 次の文章の   を埋めて、次の導関数を求めよ。  
Fill in the blanks in the following sentences to find the following derivative.

①  $\{f(x)g(x)\}'$   
$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$
$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h)g(x+h) - f(x)g(x+h)}{h} + \frac{\text{  - f(x)g(x)}{h} \right\}$$
$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h) - f(x)}{h} \right\} g(x+h) + \lim_{h \rightarrow 0} f(x) \left\{ \frac{\text{  - g(x)}{h} \right\}$$
$$= \text{ }$$

②  $\left\{ \frac{1}{g(x)} \right\}'$   
$$= \lim_{h \rightarrow 0} \frac{\frac{1}{g(x+h)} - \frac{1}{g(x)}}{h}$$
$$= \lim_{h \rightarrow 0} \frac{\text{  - \text{ }}{h \times g(x+h)g(x)}$$
$$= \lim_{h \rightarrow 0} \left\{ - \frac{\text{  - \text{ }}{h} \times \frac{1}{g(x+h)g(x)} \right\}$$
$$= \text{ }$$

③  $\left\{ \frac{f(x)}{g(x)} \right\}'$   
$$= \left\{ f(x) \times \frac{1}{g(x)} \right\}'$$
$$= f'(x) \times \text{ } + f(x) \times \left\{ \text{ } \right\}'$$
$$= \text{ }$$

2. 次の関数を微分せよ。 Differentiate the following function.

れいだい 例題	もんだい 問題
① $y = x^3$ $y' = 3x^{3-1}$ $= 3x^2$	① $y = x^2$
② $y = \frac{1}{x^3} = x^{-3}$ $y' = -3x^{-3-1}$ $= -3x^{-4}$ $= -\frac{3}{x^4}$	② $y = \frac{1}{x^2} = x^{-2}$
③ $y = (x-1)(x^4+1)$ $y' = (x-1)'(x^4+1) + (x-1)(x^4+1)'$ $= 1 \times (x^4+1) + (x-1) \times 4x^3$ $= 5x^4 - 4x^3 + 1$	③ $y = (x+1)(x^2+1)$
④ $y = \frac{1}{x+2}$ $y' = - \frac{(x+2)'}{(x+2)^2}$ $= - \frac{1}{(x+2)^2}$	④ $y = \frac{1}{x+5}$
⑤ $y = \frac{1}{x^2+1}$ $y' = - \frac{(x^2+1)'}{(x^2+1)^2}$ $= - \frac{2x}{(x^2+1)^2}$	⑤ $y = \frac{1}{x^2+x}$

1. 次の文章の   を埋めて、次の導関数を求めよ。  
Fill in the blanks in the following sentences to find the following derivative.

①  $\{f(x)g(x)\}'$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h)g(x+h) - \span style="border: 1px dashed #00FFFF; padding: 2px 10px;"> }{h} \right.$$

$$\left. \frac{f(x)g(x+h) - f(x)g(x)}{h} \right\}$$

$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h) - f(h)}{h} \right\} g(x+h) +$$

$$\lim_{h \rightarrow 0} f(x) \left\{ \frac{\span style="border: 1px dashed #00FFFF; padding: 2px 10px;">  - g(x)}{h} \right\}$$

$$= \span style="border: 1px dashed #00FFFF; padding: 10px 100px;">$$

②  $\left\{ \frac{f(x)}{g(x)} \right\}'$

$$= \lim_{h \rightarrow 0} \frac{\frac{f(x+h)}{g(x+h)} - \span style="border: 1px dashed #00FFFF; padding: 2px 10px;"> }{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) \span style="border: 1px dashed #00FFFF; padding: 2px 10px;">  - f(x) \span style="border: 1px dashed #00FFFF; padding: 2px 10px;"> }{h \times g(x+h) \times g(x)}$$

$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h)g(x) - f(x)g(x)}{h \times g(x+h) \times g(x)} - \right.$$

$$\left. \frac{f(x)g(x+h) - \span style="border: 1px dashed #00FFFF; padding: 2px 10px;"> }{h \times g(x+h) \times g(x)} \right\}$$

$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h) - f(h)}{h} \times \frac{\span style="border: 1px dashed #00FFFF; padding: 2px 10px;"> }{g(x+h) \times g(x)} \right\} -$$

$$\lim_{h \rightarrow 0} \left\{ \frac{g(x+h) - g(h)}{h} \times \frac{\span style="border: 1px dashed #00FFFF; padding: 2px 10px;"> }{g(x+h) \times g(x)} \right\}$$

$$= \span style="border: 1px dashed #00FFFF; padding: 10px 100px;">$$

2. 商の導関数の公式  $\left\{ \frac{f(x)}{g(x)} \right\}'$  において  $f(x) = 1$  と  
して  $\left\{ \frac{1}{g(x)} \right\}'$  を求めよ。  
In the formula for the derivative of the quotient, find  $\{1/g(x)\}'$  with  $f(x) = 1$ .

$$\left\{ \frac{1}{g(x)} \right\}' = \frac{\span style="border: 1px solid #000000; padding: 2px 10px;"> }{\{g(x)\}^2} = \frac{\span style="border: 1px solid #000000; padding: 2px 10px;"> }{\{g(x)\}^2}$$

3. 次の関数を微分せよ。 Differentiate the next function.

例題 れいだい	問題 もんだい
<div>① <math>y = (x^2 - 1)(x^2 + 1)</math></div> <div><math>y' = (x^2 - 1)'(x^2 + 1) +</math></div> <div><math>(x^2 - 1)(x^2 + 1)'</math></div> <div><math>= 2x \times (x^2 + 1) +</math></div> <div><math>(x^2 - 1) \times 2x</math></div> <div><math>= 4x^3</math></div>	<div>① <math>y = (x + 2)(x - 2)</math></div>
<div>② <math>y = \frac{1}{x} = x^{-1}</math></div> <div><math>y' = -1x^{-1-1}</math></div> <div><math>= -x^{-2}</math></div> <div><math>= -\frac{1}{x^2}</math></div>	<div>② <math>y = \frac{1}{x^4} = x^{-4}</math></div>
<div>③ <math>y = \frac{1}{x - 2}</math></div> <div><math>y' = -\frac{(x - 2)'}{(x - 2)^2}</math></div> <div><math>= -\frac{1}{(x - 2)^2}</math></div>	<div>③ <math>y = \frac{1}{x - 3}</math></div>
<div>④ <math>y = \frac{1}{3x + 1}</math></div> <div><math>y' = -\frac{(3x + 1)'}{(3x + 1)^2}</math></div> <div><math>= -\frac{3}{(3x + 1)^2}</math></div>	<div>④ <math>y = \frac{1}{2x + 3}</math></div>

1. 次の文章の   を埋めて、次の導関数を求めよ。  
Fill in the blanks in the following sentences to find the following derivative.

①  $\{f(x)g(x)\}'$   
$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$
$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h)g(x+h) - \text{ }}{h} + \right.$$
$$\left. \frac{\text{ } - f(x)g(x)}{h} \right\}$$
$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h) - f(x)}{h} \right\} \times \text{ } +$$
$$\lim_{h \rightarrow 0} \text{ } \times \left\{ \frac{g(x+h) - g(x)}{h} \right\}$$
$$= \text{ }$$

②  $\left\{ \frac{1}{g(x)} \right\}'$   
$$= \lim_{h \rightarrow 0} \frac{\frac{1}{g(x+h)} - \frac{1}{g(x)}}{h}$$
$$= \lim_{h \rightarrow 0} \frac{g(x) - g(x+h)}{\text{ }}$$
$$= \lim_{h \rightarrow 0} \left\{ - \frac{g(x+h) - g(x)}{h} \times \frac{1}{\text{ }} \right\}$$
$$= \text{ }$$

③  $\left\{ \frac{f(x)}{g(x)} \right\}'$   
$$= \left\{ f(x) \times \frac{1}{g(x)} \right\}'$$
$$= \text{ } \times \frac{1}{g(x)} + \text{ } \times \left\{ \frac{1}{g(x)} \right\}'$$
$$= \text{ }$$

2. 次の関数を微分せよ。

例題 れいだい	問題 もんだい
① $y = x^4$ $y' = 4x^{4-1}$ $= 4x^3$	① $y = x^5$
② $y = \frac{1}{x^4} = x^{-4}$ $y' = -4x^{-4-1}$ $= -4x^{-5}$ $= -\frac{4}{x^5}$	② $y = \frac{1}{x^5} = x^{-5}$
③ $y = (x^3 - 1)(x^3 + 1)$ $y' = (x^3 - 1)'(x^3 + 1) +$ $(x^3 - 1)(x^3 + 1)'$ $= 3x^2 \times (x^3 + 1) +$ $(x^3 - 1) \times 3x^2$ $= 6x^5$	③ $y = (x^2 + 1)(x^2 - 1)$
④ $y = \frac{1}{2x + 1}$ $y' = - \frac{(2x + 1)'}{(2x + 1)^2}$ $= - \frac{2}{(2x + 1)^2}$	④ $y = \frac{1}{3x + 1}$
⑤ $y = \frac{1}{x^2 + 2x}$ $y' = - \frac{(x^2 + 2x)'}{(x^2 + 2x)^2}$ $= - \frac{2x + 2}{(x^2 + 2x)^2}$	⑤ $y = \frac{1}{x^2 + x}$

1. 関数  $f(x) = x^n$  を二項定理を用いて、微分せよ。  
Differentiate the function  $f(x) = x^n$  using the binomial theorem.

二項定理より

$$(x + h)^n = {}_nC_0 x^n + {}_nC_1 x^{n-1} h + \cdots + {}_nC_n h^n$$

$$(x + h)^n - x^n = \boxed{\hspace{10em}}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x + h)^n - x^n}{h}$$

$$= \lim_{h \rightarrow 0} \left( \boxed{\hspace{10em}} \right)$$

$$= \boxed{{}_nC_1} = \boxed{\hspace{2em}}$$

2. 微分可能な関数  $f(x)$ ,  $g(x)$  について、次の公式を証明せよ。  
Prove the following formula for the differentiable functions  $f(x)$  and  $g(x)$ .

例題  $\{kf(x)\}' = kf'(x)$

$$\{kf(x)\}' = \lim_{h \rightarrow 0} \frac{kf(x+h) - kf(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \times k = kf'(x)$$

問題  $\{f(x) + g(x)\}' = f'(x) + g'(x)$

3. 次の文章の  $\boxed{\hspace{2em}}$  を埋めて、積の微分公式を完成せよ。  
Complete the differential formula for products by filling in the following sentences.

$$\{f(x)g(x)\}'$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h)g(x+h) - \boxed{\hspace{2em}}}{h} + \right.$$

$$\left. \frac{\boxed{\hspace{2em}} - f(x)g(x)}{h} \right\}$$

$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h) - f(x)}{h} \right\} \boxed{\hspace{2em}}$$

$$\lim_{h \rightarrow 0} \boxed{\hspace{2em}} \left\{ \frac{g(x+h) - g(x)}{h} \right\}$$

$$= \boxed{\hspace{10em}}$$

4. 次の関数  $f(x)$  を積の微分公式を用いて微分せよ。  
Differentiate the next function  $f(x)$  using the product differential formula.

例題①  $f(x) = (x + 3)^2$

$$f(x) = (x + 3)(x + 3) \text{ より}$$

$$f'(x) = (x + 3)'(x + 3) + (x + 3)(x + 3)'$$

$$= 2(x + 3)'(x + 3) = 2(x + 3)$$

問題①  $f(x) = (x + 4)^2$

例題②  $f(x) = (x + 1)(x + 3)$

$$f'(x) = (x + 1)'(x + 3) + (x + 1)(x + 3)'$$

$$= (x + 3) + (x + 1) = 2x + 4$$

$$= 2x + 4$$

問題②  $f(x) = (x + 1)(x + 4)$

例題③  $f(x) = (x + 3)(x^2 + 6x + 9)$

$$f'(x) = (x + 3)'(x^2 + 6x + 9) + (x + 3)(x^2 + 6x + 9)'$$

$$= (x^2 + 6x + 9) + (x + 3)(2x + 6)$$

$$= x^2 + 6x + 9 + 2x^2 + 12x + 18$$

$$= 3x^2 + 18x + 27$$

問題③  $f(x) = (x + 2)(x^2 + 4x + 4)$

1. 次の関数  $f(x)$  の導関数  $f'(x)$  を定義に従って求めよ。  
Find the derivative  $f'(x)$  of the next function  $f(x)$  according to the definition.

3. 次の関数  $f(x)$  を積の微分公式を用いて微分せよ。  
Differentiate the next function  $f(x)$  using the product differential formula.

例題

$$f(x)=(x+1)^2$$
$$\begin{aligned} f(x+h)-f(x) &= \{(x+h)+1\}^2-(x+1)^2 \\ &= (x+h)^2+2(x+h)+1-(x^2+2x+1) \\ &= x^2+2hx+h^2+2x+2h+1-x^2-2x-1 \\ &= h^2+2hx+2h \\ f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^2+2hx+2h}{h} = 2x+2 \end{aligned}$$

問題①

$$f(x)=(x+2)^2$$

問題②

$$f(x)=(x+3)^2$$

例題①

$$f(x)=(x+1)^2$$
$$f(x)=(x+1)(x+1) \text{ より}$$
$$\begin{aligned} f'(x) &= (x+1)'(x+1)+(x+1)(x+1)' \\ &= 2(x+1)'(x+1) = 2(x+1) \end{aligned}$$

問題①

$$f(x)=(x+2)^2$$

例題②

$$f(x)=(x+1)(x+2)$$
$$\begin{aligned} f'(x) &= (x+1)'(x+2)+(x+1)(x+2)' \\ &= (x+2)+(x+1) = 2x+3 \end{aligned}$$

問題②

$$f(x)=(x+2)(x+3)$$

例題③

$$f(x)=(x+2)(x^2-2x+4)$$
$$\begin{aligned} f'(x) &= (x+2)'(x^2-2x+4)+(x+2)(x^2-2x+4)' \\ &= (x^2-2x+4)+(x+2)(2x-2) \\ &= x^2-2x+4+2x^2+2x-4 = 3x^2 \end{aligned}$$

問題③

$$f(x)=(x+3)(x^2-3x+9)$$

2. 次の文章の  を埋めて、積の微分公式を完成せよ。  
Complete the differential formula for products by filling in the following sentences.

$$\{f(x)g(x)\}'$$
$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$
$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h)g(x+h)-\phantom{f(x+h)g(x+h)}}{h} + \right.$$
$$\left. \frac{\phantom{f(x+h)g(x+h)}-f(x)g(x)}{h} \right\}$$
$$= \lim_{h \rightarrow 0} \left\{ \frac{f(x+h)-f(x)}{h} \right\} \phantom{f(x+h)g(x+h)} +$$
$$\lim_{h \rightarrow 0} \phantom{f(x+h)g(x+h)} \left\{ \frac{g(x+h)-g(x)}{h} \right\}$$
$$= \phantom{f(x+h)g(x+h)}$$

1. 次の関数  $f(x)$  の導関数  $f'(x)$  を定義に従って求めよ。  
Find the derivative  $f'(x)$  of the following function  $f(x)$  according to the definition.

れいだい

例題

$$f(x) = \frac{1}{x-1}$$
$$f(x+h) - f(x) = \frac{1}{(x+h)-1} - \frac{1}{x-1}$$
$$= \frac{x-1}{(x+h-1)(x-1)} - \frac{x+h-1}{(x+h-1)(x-1)}$$
$$= \frac{-h}{(x+h-1)(x-1)}$$
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \rightarrow 0} \frac{-h}{(x+h-1)(x-1)} \times \frac{1}{h} = \frac{-1}{(x-1)^2}$$

もんだい

問題

$$f(x) = \frac{1}{x-2}$$

2. 次の文章の   を埋めて、商の微分公式を完成せよ。  
Fill to the following sentences to complete the quotient differential formula.

$$\left\{ \frac{1}{g(x)} \right\}'$$
$$= \lim_{h \rightarrow 0} \frac{\frac{1}{g(x+h)} - \frac{1}{g(x)}}{h}$$
$$= \lim_{h \rightarrow 0} \frac{\frac{\text{ } - \text{ }}{h \times g(x+h) g(x)}}{h}$$
$$= \lim_{h \rightarrow 0} \left\{ - \frac{\frac{\text{ } - \text{ }}{h}}{g(x+h) g(x)} \times \frac{1}{g(x+h) g(x)} \right\}$$
$$= \frac{\text{ } }{\text{ } }$$

3. 積の微分公式を利用して、商の微分公式を完成せよ。  
Complete the quotient differential formula using the product differential formula.

$$\left\{ \frac{f(x)}{g(x)} \right\}'$$
$$= \left\{ f(x) \times \frac{1}{g(x)} \right\}'$$
$$= f'(x) \times \text{ } + f(x) \times \left\{ \text{ } \right\}'$$
$$= \text{ }$$

4. 次の関数  $f(x)$  を微分せよ。 Differentiate the following function  $f(x)$ .

れいだい

例題①

$$f(x) = \frac{1}{x+3}$$
$$f'(x) = - \frac{(x+3)'}{(x+3)^2} = - \frac{1}{(x+3)^2}$$

もんだい

問題①

$$f(x) = \frac{1}{x+5}$$

れいだい

例題②

$$f(x) = \frac{1}{x^2+1}$$
$$f'(x) = - \frac{(x^2+1)'}{(x^2+1)^2} = - \frac{2x}{(x^2+1)^2}$$

もんだい

問題②

$$f(x) = \frac{1}{x^2-4}$$

れいだい

例題③

$$f(x) = \frac{x+1}{x^2+1}$$
$$f'(x) = \frac{(x+1)'(x^2+1) - (x+1)(x^2+1)'}{(x^2+1)^2}$$
$$= \frac{x^2+1-2x^2-2x}{(x^2+1)^2}$$
$$= \frac{-x^2-2x+1}{(x^2+1)^2}$$

もんだい

問題③

$$f(x) = \frac{x+3}{x^2+2}$$

