

数学Ⅱ 導関数と微分 課題

1. 次の関数を定義にしたがって微分せよ。

Differentiate the following function according to its definition.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

例題① $f(x) = 2x$

$$f(x+h) = 2(x+h) = 2x + 2h$$

$$f(x+h) - f(x) = (2x + 2h) - (2x) = 2h$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2h}{h} = 2$$

問題① $f(x) = 3x$

例題② $f(x) = 2x^2$

$$f(x+h) = 2(x+h)^2$$

$$= 2 (x^2 + 2 x h + h^2)$$

$$= 2 \mathbf{x}^2 + 4 \mathbf{x} h + 2 h^2$$

$$\begin{aligned} f(x+h)-f(x) &= (2x^2+4xh+2h^2)-(2x^2) \\ &= 4xh+2h^2 \end{aligned}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h} \\ &= \lim_{h \rightarrow 0} (4x + 2h) = 4x \end{aligned}$$

問題② $f(x) = 3x^2$

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2. 次の関数を微分せよ。

Differentiate the following function

れいだい 例題	もんだい 問題
$\textcircled{1} y = x^2 + 2x + 1$ $y' = (x^2 + 2x + 1)'$ $= (x^2)' + (2x)' + (1)'$ $= (x^2)' + 2(x)' + (1)'$ $= 2x + 2 \times 1 + 0$ $= 2x + 2$ $\textcircled{2} y = (x + 1)(x + 2)$ $= x^2 + 3x + 2$ $y' = 2x + 3$	$\textcircled{1} y = x^2 - 2x + 1$ $\textcircled{2} y = (x + 1)(x + 3)$

3. 次の関数について、与えられた x の値における微分係数を求めよ。

Find the differential coefficient of the following function at the given value of x.

れい だい 例題	もん だい 問題
$f(x) = x^2 - 4x \quad (x = 1)$ $f'(x) = 2x - 4$ $f'(1) = 2 \times 1 - 4$ $= -2$	$f(x) = x^2 + 3x \quad (x = 1)$

4. 次の関数の接線の方程式を求めよ。

Find the equation of the tangent to the following function.

$y = f(x)$ 上の点 $(a, f(a))$ を通る接線の方程式は

$$y - f(a) = f'(a)(x - a)$$

<p>例題</p> <p>$y = x^2 + 2x$ 上の点 $(1, 3)$</p> <p>$f(x) = x^2 + 2x$ とおくと $f'(x) = 2x + 2$</p> <p>$f'(1) = 2 \times 1 + 2 = 4$ より</p> <p>$y - 3 = 4(x - 1)$</p> <p>$\therefore y = 4x - 1$</p>	
<p>問題</p> <p>$y = x^2 - 2x$ 上の点 $(1, -1)$</p>	

1. 次の関数を定義にしたがって微分せよ。
Differentiate the following function according to its definition.

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

例題① $f(x) = 4x$

$f(x+h) = 4(x+h) = 4x + 4h$

$f(x+h) - f(x) = (4x + 4h) - (4x) = 4h$

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{4h}{h} = 4$

問題① $f(x) = 5x$

例題② $f(x) = -2x^2$

$f(x+h) = -2(x+h)^2$

$= -2(x^2 + 2xh + h^2)$

$= -2x^2 - 4xh - 2h^2$

$f(x+h) - f(x) = (-2x^2 - 4xh - 2h^2) - (-2x^2)$

$= -4xh - 2h^2$

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$

$= \lim_{h \rightarrow 0} (-4x - 2h) = -4x$

問題② $f(x) = -3x^2$

2. 次の関数を微分せよ。
Differentiate the following function

例題	問題
① $y = x^2 + 4x + 4$ $y' = (x^2 + 4x + 4)'$ $= (x^2)' + (4x)' + (4)'$ $= (x^2)' + 4(x)' + (4)'$ $= 2x + 4 \times 1 + 0$ $= 2x + 4$	① $y = x^2 + 6x + 9$
② $y = (x + 2)(x + 3)$ $= x^2 + 5x + 6$ $y' = 2x + 5$	② $y = (x + 3)(x + 4)$

3. 次の関数について、与えられたxの値における微分係数を求めよ。
Find the differential coefficient of the following function at the given value of x.

例題	問題
$f(x) = x^2 + 4x \quad (x = 1)$ $f'(x) = 2x + 4$ $f'(1) = 2 \times 1 + 4$ $= 6$	$f(x) = x^2 + 6x \quad (x = 1)$

4. 次の関数の接線の方程式を求めよ。
Find the equation of the tangent to the following function.

$y = f(x)$ 上の点 $(a, f(a))$ を通る接線の方程式は
 $y - f(a) = f'(a)(x - a)$

例題 $y = x^2 + 2x$ 上の点 $(2, 8)$ $f(x) = x^2 + 2x$ とおくと $f'(x) = 2x + 2$ $f'(2) = 2 \times 2 + 2 = 6$ より $y - 8 = 6(x - 2)$ $\therefore y = 6x - 4$	問題 $y = x^2 - 2x$ 上の点 $(2, 0)$
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1. 次の関数を定義にしたがって微分せよ。
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例題① $f(x) = 2x + 1$

$f(x+h) = 2(x+h) + 1 = 2x + 2h + 1$

$f(x+h) - f(x) = (2x + 2h + 1) - (2x + 1) = 2h$

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2h}{h} = 2$

問題① $f(x) = 3x - 1$

例題② $f(x) = x^2 + x$

$f(x+h) = (x+h)^2 + (x+h)$
 $= (x^2 + 2xh + h^2) + (x+h)$
 $= x^2 + 2xh + x + h^2 + h$

$f(x+h) - f(x)$
 $= (x^2 + 2xh + x + h^2 + h) - (x^2 + x)$
 $= 2xh + h^2 + h$

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
 $= \lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{h}$
 $= \lim_{h \rightarrow 0} (2x + 1) = 2x + 1$

問題② $f(x) = x^2 + 2x$

2. 次の関数を微分せよ。
Differentiate the following function

例題	問題
① $y = x^2 - 4x + 3$ $y' = (x^2 - 4x + 3)'$ $= (x^2)' + (-4x)' + (3)'$ $= (x^2)' - 4(x)' + (3)'$ $= 2x - 4 \times 1 + 0$ $= 2x - 4$	① $y = x^2 - 3x + 2$
② $y = (x + 5)(x - 1)$ $= x^2 + 4x - 5$ $y' = 2x + 4$	② $y = (x + 3)(x - 1)$

3. 次の関数について、与えられたxの値における微分係数を求めよ。
Find the differential coefficient of the following function at the given value of x.

例題	問題
$f(x) = x^2 - 3x \quad (x = 1)$ $f'(x) = 2x - 3$ $f'(1) = 2 \times 1 - 3$ $= -1$	$f(x) = x^2 - 2x \quad (x = 1)$

4. 次の関数の接線の方程式を求めよ。
Find the equation of the tangent to the following function.

$y = f(x)$ 上の点 $(a, f(a))$ を通る接線の方程式は
 $y - f(a) = f'(a)(x - a)$

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